Chance and Randomness

Random: Having no definite aim or purpose; not sent or guided in a particular direction; made, done, occurring, etc., without method or conscious choice; haphazard. (Oxford English Dictionary)

Meaning of randomness to a scientist: A random process is a process whose outcomes do not follow a deterministic pattern, but follow an evolution described by probability distributions.

Weather is a random process. It is difficult to predict the weather a few days ahead but weather follows a probability distribution in that we know broad patterns that the weather always follows -- hot in the summer and cold in the winter, for example.
Probability

Tossing a coin: Equal probability of getting heads or of getting tails:

\[ P(\text{tails}) = 0.5 \quad P(\text{heads}) = 0.5 \]

What’s the probability of getting two tails in a row?

\[ P(\text{tails})_1 \times P(\text{tails})_2 = 0.5 \times 0.5 = 0.25 \]

Since one toss of a coin doesn’t affect the next one, these are “independent events” and they should also be identical events with equal chance of heads or tails. Then the probabilities for each separate event are multiplied together.
Things that come in individual units and are not certain to occur or not to occur are subject to “counting statistics.” These statistics are made simpler if the individual units are identical, and independent of each other. Our coin toss is an illustration.

As an example, if you toss a coin 20 times (or 20 coins all together), you would “expect” that 10 of the tosses would come up heads. But that would not happen every time. However, if you repeated the 20 tosses many, many times and kept track, the average should be 10. Can we make a prediction of how many times we would get 20? 19? 18?

Think of the probabilities. There is only one way to get 20 heads – all tosses come up heads. The probability of that is \(0.5 \times 0.5 \times 0.5 \times 0.5 \ldots = 0.5^{20} = 0.000001\)

There are 20 ways to get 19 heads – any one of the tosses could come up tails. Therefore, the probability of 19 is 20 times larger than the probability of 20.

This type of logic and some mathematics leads to the result that 2/3 of the time we should get a number of heads between [the average minus the square root of the average] and [the average plus the square root of the average].

In our case, the average is 10 and \(\sqrt{10} = 3.16\), so these numbers are \(10 - 3.16 = 6.84\), while \(10 + 3.16 = 13.16\). Thus, 2/3 of the time in the coin toss we expect to get a number of heads between 6.84 and 13.16 (or, let’s say, between 7 and 13).
Okay.. So how to get std dev?
Rule of thumb

• If the average is $M$
• Then the uncertainty is $\sqrt{M}$
• $2/3$ of your measurement are within $M \pm \sqrt{M}$

Example: toss a coin 100 times for many many many times --> average = 50 heads --> uncertainty is $\sqrt{50} \approx 7$ --> about 66% of your experiments will have 43 to 57 heads
Some examples

Consider a basketball player who on average makes 50% of her free throws. If she plays a game where she gets to the foul line 8 times and she makes 6 of them, has she suddenly gotten better?

I had trouble balancing my checkbook but got pretty close. The bank said I had $123 and I came up with $133. Since the square root of 123 is just over 11, I think my answer is just as likely to be correct as theirs is, but they for some reason don’t agree. Who is right, and do they owe me $10???

The Gallup Poll sampled 1000 people. What is the margin of error for their survey?

At the end of September a poll of 1000 people showed that 51% approved and 48% disapproved of President Obama’s performance, while now 47% approve and 52% disapprove. Does this show that he is getting less popular?
Radioactive decay

1. Become other elements by emitting particles
   - **Alpha decay**: emit alpha particle (helium)
   - **Beta decay**: emit electron or positron

2. Emit photon to be stable (element unchanged)
   - **Gamma decay**: emit gamma ray

• Radioactive decays are an example of a random process subject to $\sqrt{\text{Counts}}$ errors
Notation

Mass number (\# protons plus \# neutrons) \rightarrow 12

Atomic number (\# protons) \rightarrow 6

Atomic weight \rightarrow 12.011

Note: Sometimes written as C^{12}
Isotopes: variants of an element; same number of protons; differs in neutron number
Alpha decay

Parent nucleus → Daughter nucleus

$^4_2$He → Alpha particle
Gamma Decay

Before

After

60 Co
27

60 Co
27

\( \gamma \)
Decay Types

Alpha particles (He atom nuclei) are stopped by paper.

β  Particles (electrons) are stopped by thin sheets of metal.

γ  Particles (high energy photons) need thick barriers of dense metals like lead.
Counting Statistics and Half-Life

So due to the statistics in a counting experiment, lots of samples will yield a result closer to the true average.

Both sets of boxes have the same radioactive element present but the left hand boxes have only 4 atoms per box while the right hand boxes have 400 per box.

<table>
<thead>
<tr>
<th>Number of half-lives elapsed</th>
<th>Fraction remaining</th>
<th>Percentage remaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/1</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>1/2</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>1/4</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>1/8</td>
<td>12.5</td>
</tr>
<tr>
<td>4</td>
<td>1/16</td>
<td>6.25</td>
</tr>
</tbody>
</table>
Application: Carbon dating

- $^{14}$C: unstable isotope of $^{12}$C
- $^{14}$C intake by breathing, drinking, eating...
- When dead, no more $^{14}$C intake
- Every 5730 years, half of $^{14}$C becomes nitrogen
  - Beta decay
    - 5730 yr is the “Half life” of $^{14}$C
- Know the age by measuring the amount of $^{14}$C remaining
Cosmic rays bombard upper atmosphere...

producing fast moving neutrons.

These neutrons collide with atmospheric nitrogen atoms...

producing radioactive carbon-14 ($^{14}\text{C}$)
At death, carbon intake stops!
Decay of Carbon-14

Carbon-14 remaining (%) vs. Number of half-lives (1 half-life = 5730 years)

- 10.0 g at 0 half-lives
- 5.00 g at 1 half-life
- 2.50 g at 2 half-lives
- 1.25 g at 3 half-lives

Graph shows the exponential decay of Carbon-14 over time.
We will do a lecture demonstration of the radioactive decay for Protactinium.

\[
\begin{align*}
\text{Protactinium-234 (Pa-234)} & \rightarrow \alpha \quad \text{U-238 (24.1 d)} \\
\text{Protactinium-234m (Pa-234m)} & \rightarrow \beta^- \quad (100\%) \\
\text{Protactinium-234} & \rightarrow \gamma \quad (0.15\%) \\
\text{Protactinium-234} & \rightarrow \beta^- \quad (99.85\%) \\
\end{align*}
\]

Which of these two decay types does our geiger counter see?
Sample Data

![Graph showing counts per minute over time with annotations for background from source and level without the room at all.]

- Background from source
- Use 109 for Run 1
- Use 112 for Run 2

Counts per minute vs. Time (minutes)
## Filling in the Data Table

<table>
<thead>
<tr>
<th>Clock Time</th>
<th>Elapsed Time in sec</th>
<th>Counts per minute</th>
<th>Background counts / min</th>
<th>Net Counts/min</th>
<th>Log (net counts/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1 Run 2</td>
<td>Run 1 Run 2</td>
<td>Run 1 Run 2</td>
<td>Run 1 Run 2</td>
<td>Run 1 Run 2</td>
<td>Run 1 Run 2</td>
</tr>
<tr>
<td>8:14:52</td>
<td>0 0</td>
<td>409</td>
<td>109 112</td>
<td>300</td>
<td>2.48</td>
</tr>
<tr>
<td>8:15:03</td>
<td>11</td>
<td>361</td>
<td>” ”</td>
<td>252</td>
<td>2.40</td>
</tr>
<tr>
<td>8:15:14</td>
<td>22</td>
<td>367</td>
<td>” ”</td>
<td>258</td>
<td>2.41</td>
</tr>
</tbody>
</table>

Net Counts/min = Counts/min – Background counts / min
Data Table Format

Data for Runs 1 and 2 will be posted on the class home page.

<table>
<thead>
<tr>
<th>Time</th>
<th>Counts/minute</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:14:52AM</td>
<td>409</td>
</tr>
<tr>
<td>8:15:03AM</td>
<td>361</td>
</tr>
<tr>
<td>8:15:14AM</td>
<td>367</td>
</tr>
<tr>
<td>8:15:25AM</td>
<td>367</td>
</tr>
<tr>
<td>8:15:36AM</td>
<td>319</td>
</tr>
<tr>
<td>8:15:47AM</td>
<td>331</td>
</tr>
<tr>
<td>8:15:58AM</td>
<td>337</td>
</tr>
<tr>
<td>8:16:09AM</td>
<td>259</td>
</tr>
<tr>
<td>8:16:20AM</td>
<td>217</td>
</tr>
<tr>
<td>8:16:31AM</td>
<td>271</td>
</tr>
<tr>
<td>8:16:42AM</td>
<td>271</td>
</tr>
<tr>
<td>8:16:53AM</td>
<td>313</td>
</tr>
<tr>
<td>8:17:04AM</td>
<td>271</td>
</tr>
<tr>
<td>8:17:14AM</td>
<td>217</td>
</tr>
</tbody>
</table>