A perfect linear polarizer transmits one direction of vibration while completely blocking the orthogonal. Hence, viewed through a linear polarizer, an intensity (or flux) measured over a finite time interval is

\[ I_x = \langle E_0^2 \cos^2 \theta [\cos \omega t \cos \varphi_x - \sin \omega t \sin \varphi_x]^2 \rangle \]

\[ I_y = \langle E_0^2 \sin^2 \theta [\cos \omega t \cos \varphi_y - \sin \omega t \sin \varphi_y]^2 \rangle \]

If no circular polarization, \( \varphi_x = \varphi_y \). Then,

\[ I_x - I_y = \langle E_0^2 \cos^2 \theta [...]^2 \rangle - \langle E_0^2 \sin^2 \theta [...]^2 \rangle \]

\[ = \langle \cos 2\theta \rangle \langle E_0^2 [...]^2 \rangle \]

\[ I_x + I_y = \langle E_0^2 \cos^2 \theta [...]^2 \rangle + \langle E_0^2 \sin^2 \theta [...]^2 \rangle \]

\[ = \langle E_0^2 [...]^2 \rangle \]
Measuring the Stokes Vector

• So, ratio
\[
\frac{(I_x - I_y)}{(I_x + I_y)} = \langle \cos 2\theta \rangle = q = Q/I \quad \text{(in absence of circular polarization)}
\]

• Rotating coordinate system 45° switches \(\cos 2\theta \rightarrow \sin 2\theta\), so second set of 2 measurements yields \(u = U/I\). (see def’n of Stokes parameters when \(\chi = 0\)).

• Then, linear polarization \(P\) and position angle \(\theta\) (measured N \(\rightarrow\) E) are defined
\[
P^2 = q^2 + u^2
\]
and
\[
\theta = \frac{1}{2} \arctan \left( \frac{u}{q} \right)
\]
Motivates use of so-called “q-u”, or “2θ” diagram.

Note: $P > 0$ (relevant for error analysis)

\[ P^2 = q^2 + u^2 \]

and
\[ \theta = \frac{1}{2} \arctan \left( \frac{u}{q} \right) \]
Linear Polarization in Nature

- Blackbody emission in absence of strong $B$-field is unpolarized.
- Scattering of light off small particles (electrons or dust grains). $P$ can approach 100% and $q$ almost always $\perp$ scattering plane.
- Reflection of light off macroscopic objects (e.g., planetary surfaces). $P$ typ. $< 3\%$.
- Emission by electrons in magnetic field with transverse component.
  - Synchrotron emission (relativistic)
  - Cyclotron emission (non- or quasi-relativistic)
  - Zeeman effect
  $P \sim 100\%$ possible.
Linear Polarization by Scattering

Polarization map (left) and total light image (right) of the bipolar Egg nebula, both in near-infrared light. The nebula reflects light from an embedded star, which is evolving to become a planetary nebula + white dwarf.
Detection of Hidden Broad-Line Regions in AGN
Circular Polarization in Nature

• Measures tendency of E-vector to spiral around Poynting vector.
• Astronomical convention: \( \nu > 0 \) when E-vector rotates CCW as viewed by observer looking toward source.
• \(-1 < \nu < +1; \ |\nu| \) = fractional amount of circular polarization.

• Blackbody emission in absence of strong \( B \)-field is unpolarized.
• Produced in Nature by:
  - Electrons gyrating in magnetic field with component along line of sight (Zeeman effect, cyclotron emission). \( \nu > 50\% \) possible.
  - Scattering of linearly polarized light off particles with dimension \( a \sim \lambda \). Typically, \( \nu \ll 1\% \).
The Zeeman Effect

SDSS spectra of magnetic white dwarfs showing the Zeeman effect on the Balmer series, $B = 0$–30 MG ($MG = 10^6$ G)
Circularly Polarized Zeeman Lines from Magnetic WDs

WD 1350–090
$B_0 = +85 \text{ kG}$
Circularly Polarized Cyclotron Lines from Magnetic Accretors
Sheet Linear Polarizers

At the heart of every polarimeter is a linear polarizer. (Natural circular polarizers do not exist). Sheet polarizers use aligned, long molecules to selectively absorb one direction of vibration.

Assume:

- $t$ = unpolarized transmittance
- $l$ = “leak” of unwanted, orthogonal component

Then, if input is unpolarized, fraction transmitted is:

- Single: $\frac{1}{2} t (1+l)$
- 2 Crossed: $t^2 l$
- 2 Parallel: $\frac{1}{2} t^2 (1+l^2)$
- Pol. efficiency: $(1 - l) / (1 + l)$
Refractive Indices in Crystals

Crystals can be uniaxial (common; shown above at left) or biaxial and have negative or positive birefringence (\(n_e < n_o\) or \(n_e > n_o\)).
Birefringent Linear Polarizers

Crystal polarizers utilize different $n$ for different vibration planes.

MgF$_2$: $n_e \sim 1.38$; $n_e - n_o \sim +0.011$
2000Å – 6µm

Quartz: $n_e \sim 1.55$; $n_e - n_o \sim +0.009$
2500Å – 2.5µm

Calcite: $n_e \sim 1.66$; $n_e - n_o \sim -0.176$
3500Å – 1.8µm (Best CaCO$_3$ still found in mines/caves. But, soft and hygroscopic)

t approaches 1.0

Extinction can achieve $l < 10^{-5}$, so pol. efficiency up to 99.999%!
Fun & Games with Crystals

Nicol, Glan-Laser, Glan-Foucault

Beam displacer
Waveplates (Retarders)

- $\Delta \varphi = 2\pi \frac{t (n_o - n_e)}{\lambda_0}$.
- If $n_o - n_e = 10^{-2}$, $t \sim 0.01\text{mm}$ yields $\Delta \varphi \sim \pi/2 = \lambda/4$.

What does this do for us?

- $[\text{lin. pol } @ \theta] + [\lambda/4 \text{ retarder } @ \theta \pm 45^\circ] \Rightarrow [\pm \text{ circ. pol}]$
- $[\pm \text{ circ. pol}] + [\lambda/4 \text{ retarder } @ \theta ] \Rightarrow [\text{lin. pol } @ \theta \pm 45^\circ]$
- $[\text{lin. pol}] + [\lambda/2 \text{ retarder } @ \theta ] \Rightarrow \text{lin. pol } @ -\theta$

- So, to measure circular polarization, convert it to linear and measure the linear!
A Simple Astronomical Polarimeter for Point Sources

Problems:
- Rotate entire instrument to measure $u$ Stokes parameter.
- Orthogonal signals for each Stokes measured by different detectors.
A Modern Imaging/Spectropolarimeter

Two exposures with $\lambda/2$ rotation measure a Stokes parameter in each spectrum. Additional rotations of $\lambda/2$ measure other Stokes parameter.
Practical Polarimetry

- Refractive indices vary with $\lambda$, so must **achromatize** waveplates to work over wide $\Delta\lambda$.
- Time-modulate observations to minimize influences of variations in seeing, guiding, and detector sensitivity.
- Moonlight is very strongly polarized.
Polarimetric Error Analysis

\[ q = \frac{F_1 - F_2}{F_1 + F_2} \]

\[ \sigma^2(q) = \left( \frac{\partial q}{\partial F_1} \right)^2 \sigma^2(F_1) + \left( \frac{\partial q}{\partial F_2} \right)^2 \sigma^2(F_2) \]

Assume \( F \propto N \)

Then can show \( \sigma^2(q) = \frac{1 - q^2}{N_1 + N_2} \)

For \( q \ll 1 \),

\[ \approx \frac{1}{N_1 + N_2} \]

so \( \sigma(q) \approx \frac{1}{\sqrt{N_1 + N_2}} \)

With great care, can achieve sensitivities of \( \sigma_p \sim 0.001\% \). To do this, must detect \( >2 \times 10^{10} \) photons!

Often, \( \sigma(q) \approx \sigma(u) \approx \sigma(P) \)

Then,

\[ \sigma(\theta) \approx \frac{1}{2} \frac{\sigma(P)}{P} \]

\[ = 28.65 \frac{\sigma(P)}{P} \text{ in degrees} \]
First-order correction for bias in $P$:

$$\sigma(P) \approx \sigma(P_{\text{obs}}) \approx \sigma(q) \approx \sigma(u)$$

$$P \approx P_{\text{obs}} \sqrt{1 - \frac{\sigma^2(P_{\text{obs}})}{P^2_{\text{obs}}}}$$

Probability distribution for $\theta$ at low-S/N is non-Gaussian and problematic!

See
Simmons & Stewart, 1985, A&A, 142, 100
(and similar)
How an LCD Display Works

• Tiny (~20Å) liquid crystals only absorb and emit \perp\text{ to their length.}
• Voltage off: Crystals twisted and LCD pixel ON (left).
• Voltage on: Crystals straighten and LCD pixel OFF (right).
“Q-Switch” for Ultra-Short Light Pulses

- Pockels cell or Kerr cell operating as on/off $\lambda/4$ plate.
- Cell on: No pulse
- Cell off: Pulse