Chapter 5: Photometry and Polarimetry

5.1 Introduction

Most of what we know about astronomical sources comes from measuring their spectral energy distributions (SEDs) or from taking spectra. We can distinguish the two approaches in terms of the spectral resolution, defined as $R = \frac{\lambda}{\Delta \lambda}$, where $\lambda$ is the wavelength of observation and $\Delta \lambda$ is the range of wavelengths around $\lambda$ that are combined into a single flux measurement. Photometry refers to the procedures for measuring or comparing SEDs and is typically obtained at $R \sim 2 - 10$. It is discussed in this chapter, while spectroscopy is described in the following one.

In the optical and near infrared, nearly all the initial photometry was obtained on stars, whose SEDs are a related family of modified black bodies with relative characteristics determined primarily by a small set of free parameters (e.g., temperature, reddening, composition, surface gravity). Useful comparisons among stars can be obtained relatively easily by defining a photometric system, which is a set of response bands for the [(telescope)-(instrument optics)-(defining optical filter)-(detector)] system. Comparisons of measurements of stars with such a system, commonly called colors, can reveal their relative temperatures, reddening, and other parameters. Such comparisons are facilitated by defining a set of reference stars whose colors have been determined accurately and that can be used as transfer standards from one unknown star to another. This process is called classical stellar photometry. It does not require that the measurements be converted into physical units; all the results are relative to measurements of a network of stars. Instead, its validity depends on the stability of the photometric system and the accuracy with which it can be reproduced by other astronomers carrying out comparable measurements.

In other circumstances, it is necessary to convert measurements with a photometric system into physical units. This situation applies in general for non-stellar targets and, since such targets are dominant away from the optical and near-infrared, it is the rule for most wavelength regions. The initial steps are similar to those for classical stellar photometry; a photometric system must be defined, a network of standard sources is established, and measurements are made relative to these standard sources. However, the outputs of the standards must be known in physical units, so the intercomparisons can be expressed in such units.

Polarimetry is an additional avenue to learn about astronomical sources. The instrumental approaches stem from those used in photometry but for good performance require a high degree of specialization. They are described at the end of this chapter.

5.2 Stellar Photometry
5.2.1 Source Extraction and Measurement

To provide a specific framework for our discussion, we will assume for now that we are working with optical or infrared array data. Given a well-reduced image, as described in the preceding chapter, photometric information can be extracted by two different approaches.
Aperture photometry: a digital aperture is placed over the source image and the counts within the aperture are summed. To determine the contribution to these counts from the sky (allowing them to be subtracted), either the aperture is placed on nearby blank sky and the counts summed, or the sky level is determined over an annulus around the source position. This procedure is the same as is used with a single detector aperture photometer, but it is done in the computer after the data have been obtained (see Stetson 1987, 1990 for more information). There are a number of considerations in applying it effectively:

- The aperture is placed on the basis of a centroiding routine, such as column and row sums or central moment, or by eye. Software such as DAOFIND or can be used to locate all the sources in a field so allow efficient photometry of multiple sources in an array image.

- An aperture diameter must be defined. It is not necessary to get nearly all the signal, so long as one is measuring point sources and all of them are done in the same way. In crowded fields one must use small apertures. For faint sources, there is a relatively small aperture size that optimizes the signal to noise even with no nearby sources. One should experiment with different aperture sizes to determine which is best for any application. In the end, one will correct for the signal lost from the aperture using measurements in multiple apertures on a bright, isolated source - this factor is the "aperture correction."

- The "sky" will have deviations with a bias toward positive values (e.g., sources). You can use software to remove the obvious ones and replace them with mean surrounding values. In general, you may want to estimate sky as the mode, rather than median or mean, since the mode (most often occurring value) is most immune to bias.

Point spread function (PSF) fitting: The PSF is determined on a bright star and this profile is fitted to the images in the frame by least squares minimization. In crowded fields, the fitting can include many stars at once (e.g., DAOPHOT) to extract a best fit to the photometry of overlapping sources.

- PSF fitting is more immune to anomalies than aperture photometry (e.g., a deviant pixel due to a poorly removed cosmic ray hit will just be counted at its measured value in aperture photometry, but will generally have a modest effect on the PSF fit to the image)

- Calibration is preserved either with a PSF fit to a standard, or more commonly by comparing aperture photometry on an isolated bright source in your field with the PSF fit result.

- Obviously, PSF fitting is not appropriate for extended sources, for which one needs to use aperture photometry or an automatic program designed for extended sources. Programs are available that allow definition of arbitrarily shaped apertures to fit the profile of a source (e.g., IDP3 developed by the NICMOS project). Another approach is to conduct surface photometry within isophotes (lines of constant surface brightness). A popular alternative is to fit the source with an ellipse (or series of nested ellipses) and define the photometry within them. Sextractor (Bertin and Arnouts 1996) has been developed to identify and obtain approximate photometry of extended sources in large fields. It is not a substitute for surface photometry of well-resolved sources but is useful for analyzing survey observations.
Photometry

- Programs that do automated PSF fitting over large fields such as DAOPHOT (Stetson 1987) or DOPHOT (Mateo and Schechter 1989) are essential for many data reduction projects.

That all sounds easy. However, there can be complications, of which we postpone discussion to Section 5.3.

Assuming we have adjusted our optical system to put multiple pixels across the diameter of the image, simple aperture photometry is unlikely to extract the source signal in an optimum way. It amounts to convolving the image of the source with a “top hat” – in this case cylindrical – function, giving equal weight to all the pixels inside the aperture. That is, for well-sampled images the spatial frequencies in the signal have been filtered by the telescope; however, no filtering has been applied to the noise, which can vary freely from pixel to pixel. Each pixel will then contribute roughly an equal amount of noise, but the signal will be strongly weighted toward the brightest parts of the image. In some situations the aperture placed on the source is made relatively large as a way to capture most of the signal and improve the accuracy of the measurement, but this approach also increases the noise (from the sky) in the interest of capturing relatively small increments in the signal.

We should be able to smooth the image to suppress the noise while having little effect on the signal. As we might guess, the way to optimize the signal to noise is to convolve the signal with a filter that reproduces the expected shape of the image, i.e., the point spread function. Recalling the MTF of the PSF, this convolution suppresses the high spatial frequencies in the noise just to match the system response to a real signal. Thus, PSF fitting is an optimal way to extract a signal, particularly one detected at limited signal to noise. This result extends to identifying sources in the face of noise; it is often advantageous first to filter the entire image with a kernel given by the point spread function and only then to identify possible faint sources. Another advantage of PSF fitting is that the results are more immune to one or two anomalous signals among the pixels used for the photometry; in aperture photometry, such signals are included fully in the result, whereas the PSF fit generally minimizes their contribution.

5.2.2 Photometric Systems and Terminology

We will continue to assume that the measurements have been obtained from images. For many years, the standard photometric instrument was a single detector that measured objects through a fixed aperture at the telescope focal plane. The details of such instruments and their measurements can be found in older books on astronomical instrumentation. The methods for extracting signals differ from those for imaging data. However, once the measurements have been obtained, the following considerations are virtually identical.

If the goal is to measure stellar colors in a relative sense, then we assume data were obtained as follows:
1.) Select a detector and a suite of filters for your measurements
2.) Define some combination of reference stars that define “zero color”
3.) Measure a network of ‘standard stars’ of varying colors relative to these reference stars
4.) Measure the science targets relative to these standard stars
Assuming we do not want to fight 20 centuries of tradition, we will express the results in the magnitude system. In general, the magnitude difference, $m_1 - m_2$, between objects 1 and 2 at the same photometric band is

$$m_1 - m_2 = -2.5 \log \left( \frac{f_1}{f_2} \right)$$  \hspace{1cm} (5.1)

where $f_1$ and $f_2$ are respectively the fluxes of objects 1 and 2 in the band. Thus, the system is logarithmic, useful given the huge range of brightness of astronomical sources, and is a somewhat arcane way of describing flux ratios. To provide a universal system, the apparent magnitude, $m$, is defined as in equation (5.1) but with star 2 a “zero point” flux, that is a flux that defines the magnitude scale and is based on a star or suite of stars to provide consistent results in multiple photometric bands. In general, the zero-point-defining star may not be accessible for direct observation (it may be below the horizon, too bright, or may not exist – if the zero point is defined as a combination of fluxes from more than one star, for example). So we have:

$$m = -2.5 \log \left( \frac{f_{\text{star}}}{f_{\text{ZP}}} \right)$$  \hspace{1cm} (5.2)

or

$$m = -2.5 \log \left( \frac{f_{\text{star}}}{f_{\text{standard}}} \right) + m_{\text{standard}}$$  \hspace{1cm} (5.3)

Here $m_{\text{standard}}$ is relative to the zero point. In general, the reduced data will be quoted in "instrumental magnitudes", that is, in magnitudes relevant for comparing stars in a given dataset but without reference to any zero point. It is necessary to find what signals standards give in instrumental magnitudes to put the data on a standard scale. The magnitude of the standard star in equation (5.3) is then subsumed into a “zero point term.”

The absolute magnitude, $M$, is the magnitude a star would have at a distance of 10pc. The distance modulus is the difference between apparent and absolute magnitudes, $m - M$:

$$m - M = 5 \log \left( \frac{\text{distance in pc}}{10\text{pc}} \right) = 5 \log \left( \frac{d}{10\text{pc}} \right)$$  \hspace{1cm} (5.4)

Here $d$ is the distance of the star with magnitude $m$, and the effects on $m$ of extinction are not included (or equivalently the formulation is only useful under the assumption that $m$ is corrected for extinction).

To go further, we need to introduce a specific photometric system as an example. We select the “Arizona” $UBVRIKLMQ$ system introduced by Harold Johnson with an assist by Frank Low (and H added by Eric Becklin). The Johnson UBVR system was originally based on combinations of colored glass and the spectral responses of photomultipliers. The I band was defined by an interference filter (described in Section 6.6) that uses interference effects in multiple layers of optical materials to create a filter passband. The bands are shown in Figure 5.1; because the atmospheric transmission is good and relatively featureless across the spectral range of these bands (Figure 1.17), they could be defined for convenience. This system had the appearance of being cheaply and easily reproduced, but the detector-dependence resulted in a single “system-defining” photometer. When this photometer was no longer operable, officially the system died.
The system was extended into the infrared with interference filters. The infrared bands are defined by interference filters and have varied from observatory to observatory as a result of the availability of the filters. They are tied to the terrestrial atmospheric windows as shown in Figure 5.2.

With a photometric system, we can continue our discussion of definitions. We extend the definitions of apparent magnitude to the bands of the system, e.g., the magnitude in the B band is $m_B$. The color index, $CI$, is the difference between the magnitudes of a star in two different bands, e.g., $CI = m_B - m_V = B-V$, where the last version is frequently used. The color excess, $E$, is the difference between the observed $CI$ and the standard $CI$ for the stellar type, e.g.,

$$E_{BV} = CI_{star} - CI_{stellar\ type} \quad (5.5)$$

The bolometric magnitude, $M_{bol}$, is the stellar luminosity expressed as an absolute magnitude. Curiously, the zero point for bolometric magnitudes has been defined in more than one way; we recommend using the IAU-recommended one that $M_{bol} = 0$ at $3.055 \times 10^{28}$ W. However, variations at the level of a few percent will be found (generally without warning) in the literature. The bolometric correction, $BC$, is the correction in magnitudes that must be applied to the absolute magnitude in some band to obtain $M_{bol}$.

Johnson selected six A0 stars and averaged their colors, setting Vega close to zero magnitude (for arcane reasons related to the five other stars, Vega was not set exactly to zero) to define the zero point of the UBVRI part of the system. This philosophy was extended into the infrared, with the complication that not all the defining stars were detectable at the time in all the infrared bands; thus, the dependence on the brightest, Vega, became much greater. Extensive measurements were made in this system to establish the basic behavior of normal stars. This system is called the Arizona/Vega system of photometry and magnitudes under it are called Vega magnitudes.

Following astronomical tradition, Vega was a very bad choice for a star to define photometric systems. It has a debris disk that contributes a strong infrared excess above the photosphere, already detected with MSX (ref.) at the $\sim 3\%$ level at 10$\mu$m and rising to an order of magnitude in the far infrared. Interferometric measurements at 2$\mu$m show a small, compact disk that contributes $\sim 1\%$ to the total
Photometry flux. Vega is a pole-on rapid rotating star with a 2000K temperature differential from pole to equator. This joke nature has played on Harold and the rest of us accounts for some of the remaining discrepancies in photometric calibration.

5.3 Photometry made complicated
Like almost anything else in life, photometry can be made as complicated as you wish. We now describe some of the reasons, although there can be more. At the same time, you should approach this issue by thinking carefully about the requirements of your particular science program and only pursue the refinements to the point that is needed.

5.3.1. Corrections for atmospheric absorption
Photometry of multiple sources obtained from the ground will in general be measured along different paths through the atmosphere. Although on occasion it is possible to measure the sources of interest at nearly the same elevation and over a short time interval, in general the measurements need to be corrected for the differing amounts of atmospheric absorption. Conventionally, the measurements are all corrected to the equivalent values if the measurements were made directly overhead, through one “air mass.” The different slant paths are described in terms of the factor (> 1) by which the atmospheric path is increased.

For a plane parallel atmosphere, the air mass goes as the secant of the zenith angle, z:

\[ \text{air mass} = \sec(z) \left( 1 - 0.0012 (\sec^2 z - 1) \right) \] (5.7)

Assuming the extinction is exponential,

\[ \frac{dl_\lambda}{ds} = -\kappa_\lambda l_\lambda \] (5.8)

where \( l_\lambda \) is the intensity and \( \kappa_\lambda \) the absorption coefficient and \( s \) is the path length. Then

\[ m_\lambda(\text{am}) - m_\lambda(\text{am} = 1) = 2.5 \kappa_\lambda (\text{am} - 1) \] (5.8)

where the magnitudes are the observed values at airmass, am. Thus, observations at a range of airmasses (zenith angles) can be fitted to estimate the absorption coefficient and to correct a suite of measurements to unity airmass. In addition, the standard stars should have a broad range of color to help evaluate the differences in some of the bands as a function of atmospheric conditions.

5.3.2 Aperture corrections
Both aperture and PSF-fitting photometry require a good understanding of the point spread function. This is obvious for PSF-fitting. In the former case, it is never possible to use a sufficiently large aperture to capture all the flux from a source, and optimizing for signal to noise or in crowded fields generally leads to apertures that exclude a significant fraction of the flux. It is necessary to determine an “aperture correction” to compensate for the lost flux (except in the special case where one can be sure
that the correction is identical for all the measurements). In general, such a correction may even be needed for PSF-fitting because the sky level used in the fit may still contain faint wings of the PSF.

So far, we have assumed that the PSF is a single function for a given instrument and set of images. PSF fitting is relatively easy if this assumption is correct, as it might well be for an instrument in space (above the atmosphere) and with well-corrected optics. Even in this case, however, if the photometric band is relatively wide the PSF will vary with the spectrum of the source. For optical and infrared imaging from the ground, the seeing can change with time. A solution is to use a PSF-matching filter to adjust the differing PSFs. Calculating this filter by brute force can be clumsy. Alard and Lupton (1998) suggested that the process can be streamlined by calculating in advance a set of basis functions that can be used in linear combination to produce the PSF-matching filter. This approach is also useful when the PSF varies over the field of view. A variable PSF also makes noise estimation more difficult. A reliable procedure is to place artificial sources with the shape of the local PSF into the image and then extract and measure them to understand the real dependence of photometric accuracy on source strength.

5.3.3. Maintaining the photometric system

With traditional stellar photometry, the intercomparison of measurements depends on the ability to reproduce the spectral properties of the system and to maintain them. The original Johnson UBVRI system depended for its definition on a specific photometer - and whenever a detector failed, there was a crisis to find a replacement with the similar spectral properties and then to re-establish the system with it. By now, people have long since abandoned the idea of a defining photometer; instead, systems are used that are thought to be equivalent from the spectral properties of their optical components.

Be aware that not all “equivalent” systems are! There are a number of systems advertised as UBVRI that differ dramatically from Johnson’s original. The”I” band is particularly problematic, with center wavelengths ranging by over 0.1μm! This problem arises with many other photometric systems also. Even identical detectors and filters will have differing spectral properties if operated at different temperatures or in optical systems with different f/numbers at the position of a filter. Thus, two systems using identical detectors and filters but at different temperatures will have different spectral properties. Furthermore, the atmospheric transmission in some bands depends on atmospheric conditions. For examples, look at the ‘M’ and ‘Q’ bands for which Figure 5.2 gives transmissions for differing amounts of water vapor. The U band is similarly subject to atmospheric effects. Nonetheless, much of the foundation for photometry, particularly in the visible, rests on measurements made in a small number of standard systems, the Arizona one being an outstanding example.

To deal with the problems caused by differences in photometric systems, two systems can be reconciled through observation of many stars in both and comparing the results. A fit is generated (as a function typically of the color of the source and perhaps the air mass) that allows transformation of the actual observed values with one system to the values that would have been observed with the other. An example is shown in equations 5.9 that show how to convert JHK measurements in the older CIT system (Elias et al. 1982) to the 2MASS system.

\[
(K_S)_{2MASS} = K_{CIT} + (0.000 \pm 0.005)(J - K)_{CIT} + (-0.024 \pm 0.003)
\]

\[
(J - H)_{2MASS} = (1.076 \pm 0.010)(J - H)_{CIT} + (-0.043 \pm 0.006)
\]

\[
(J - K_S)_{2MASS} = (1.056 \pm 0.006)(J - K)_{CIT} + (-0.013 \pm 0.005)
\]
(H − K_S)_{2MASS} = (1.026 \pm 0.020)(H − K)_{CIT} + (0.028 \pm 0.005) \quad (5.9)

Here, $K_S$ is the specific 2MASS K band, which differs significantly from previous definitions of this band. The equations are in terms of the stellar colors because the differing spectral responses of the two systems affect the colors directly. The corrections can be fairly large, as shown in the example in equations (5.9). Application of the transformations can homogenize the photometric systems so long as they are being applied to normal stars, since the transformations refer to such stars. However, for demanding work, be suspicious. For example, the number of measurements suitable for computing transformations between some systems is quite small and the transformations are correspondingly uncertain. Also, photometric transformations are generally less reliable in the visible and ultraviolet, where stars have many strong absorption features, than in the infrared where their spectra are relatively similar and feature-free.

In the past, another source of error was that standard stars at widely different positions on the sky might not be intercompared to high accuracy. New homogeneous all sky databases (2MASS for NIR, Hipparcos for BV) make the situation much better, since they provide many stellar observations that are extremely homogeneous and remove a number of possible causes of uncertainty in computing transformations.

It can be difficult to reach accuracies better than the 1% level with imaging detectors used in standard ways. However, under special circumstances, imagers can do considerably better than they do for routine photometry - for example, where sources are always measured at the same positions and the images are well sampled (sometimes the images are put slightly out of focus or are moved systematically over the array to spread them over more pixels).

5.4. Physical Photometry
5.4.1 General approach

Classical stellar photometry is of little use when studying something other than a relatively normal star, since it assumes a related family of pseudo-blackbody spectra. In addition, the underlying precepts of classical photometry break down at wavelengths where stars are not readily measured.

A physical system converts the measurements into flux densities at specific wavelengths. To each filter (Figure 5.3), we assign a wavelength, $\lambda_0$. The mean wavelength is the simplest, although we will discuss alternatives later:

$$\lambda_0 = \frac{\int \lambda T(\lambda) \, d\lambda}{\int T(\lambda) \, d\lambda} \quad (5.10)$$

Figure 5.3. System transmission characteristics.
Photometry

$T(\lambda)$ is the wavelength dependent system response, called the relative spectral response function. It includes the full spectral transmittance (including filter, detector, telescope, and atmosphere) for the photons reaching the detector, convolved with the detector response. In general for the latter the detector spectral responsivity should be used, the current out of the detector as a function of wavelength and the photon power into it (see Chapter 3), since this current is the signal that we work with.

5.4.2 Absolute Calibration
One might expect that physical photometry could operate with a carefully calibrated detector and thorough understanding of the telescope and instrumental efficiency, to obtain source measurements directly in physical units. However, it turns out to be very difficult to achieve a high degree of accuracy in this way; the telescope and instrumental efficiencies are difficult to measure and are subject to change. Therefore, physical photometry is carried out relative to standard stars or other sources, just as in classical photometry. To carry out physical photometry, we conduct the same set of measurements just described for stellar photometry, including all of the corrections to unity air mass (but not to transform to a standard photometric system). We will use our fully reduced measurements to determine the flux density of sources at the effective wavelengths of our photometric bands.

The challenge is then to know the flux densities of our standard stars accurately. There are a number of possibilities to get this information.

*Direct calibrations:* One compares the signal from a calibrated blackbody reference source to those from one or more members of the standard star network. Ideally, one would use the same telescope and detector system to view both, but often the required dynamic range is too large and it is necessary to make an intermediate transfer.

*Indirect calibrations:* One can use physical arguments to estimate the calibration, such as the diameter and temperature of a source. A more sophisticated approach is to use atmospheric models for calibration stars to interpolate and extrapolate from accurate direct calibrations to other wavelengths.

*Hybrids:* The solar analog method is the most relevant hybrid approach. It uses absolute measurements of the sun, assumes other G2V stars have identical spectral energy distributions, and normalizes the solar measurements to other G2V stars at some wavelength where both have been measured, such as $m_V$.

The calibrations in the visible are largely based upon comparisons of a standard source (carefully controlled temperature and emissivity) with a bright star, usually Vega itself. Painstaking work is needed to be sure that the very different paths through the atmosphere are correctly compensated. In the infrared, there are three current approaches that yield high accuracy: 1.) measurement of calibrated sources by the MSX satellite mission and comparing the signals with standard stars. This experiment has provided the most accurate values; 2.) measurement of Mars relative to standard stars while a spacecraft orbiting Mars was making measurements in a similar pass band and in a geometry that allowed reconstructing the whole-disk flux from the planet; 3.) the Solar analog method, comparing accurate space-borne measurement of the solar output with photometry of solar-type stars.
Accurate (better than 2% estimated uncertainty) calibrations are now available for the range 0.3 through 25μm. They are usually expressed as the flux density corresponding to zero magnitude at each effective wavelength, e.g. at the ‘zero point’. Thus, accurate physical measurements can be deduced over this range of wavelengths by measuring signals relative to standard stars that are traced to the absolutely calibrated network, correcting for atmospheric extinction, and then applying the calibration directly to the measured magnitudes.

### 5.4.3 Bandpass Corrections

Although we want to quote a flux density at a specific wavelength, the signal is proportional to the convolution of the source spectrum with the instrument spectral response function (see Figure 5.4). Therefore, sources with the same flux densities at \( \lambda_0 \) but different spectral shapes may give different signals – and conversely, sources giving the same signals may not have the same flux densities at \( \lambda_0 \). We need to compensate for this effect, or the accurate absolute calibrations now available will be undermined. With knowledge of the source spectrum, one can calculate the necessary corrections by integrating it and the standard star spectra over the passband and comparing the results.

For example, any source so cold that the peak of its blackbody falls at longer wavelengths than the system passband has a spectrum rising very steeply (Wien side of blackbody) across the passband. Most of the signal will be derived from wavelengths longer than \( \lambda_0 \), and the flux density at the center of the passband can be small even with a significant net signal. A correction in the opposite direction is required when the blackbody peaks near \( \lambda_0 \). The size of the corrections scales as \((\Delta \lambda/\lambda_0)^2\), where \( \Delta \lambda \) is the filter transmission width (Figure 5.3). The correction is moderate in size for \( \Delta \lambda/\lambda_0 < 0.25 \), but for broader filters the uncertainties in the correction can dominate other uncertainties in the photometry.

### 5.4.4 Attempts to minimize bandpass corrections

![Figure 5.4. Comparison of measurements of a hot star (A0: heavy dashed line) and a power law spectrum (\( \lambda^{-0.5} \): heavy dotted line) in the J filter (heavy solid line). The convolution of the spectra with the filter transmission is shown by the light dashed and light dotted lines, respectively. The measured signal is proportional to this convolution. The input spectra (heavy lines) have been adjusted to the signals from them are equal. The dashed arrow shows where these input spectra are equal, whereas the solid arrow shows the mean wavelength of the filter.](image-url)

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Photometry

To reduce the bandpass corrections, some astronomers use alternates to $\lambda_0$. For example, there is the effective wavelength (used by IRAS and IRAC on Spitzer among others):

$$\lambda_{\text{eff}} = \frac{\int \lambda^2 T(\lambda) d\lambda}{\int \lambda T(\lambda) d\lambda} \quad (5.11)$$

This definition reduces the corrections for warm and hot objects and increases them for cold ones.

Yet another approach is embodied in the isophotal wavelength. The idea is not to adjust the measured flux density, but to adjust the wavelength of the measurement so the measured flux density applies at that wavelength. This process is mathematically equivalent to adjusting the flux density, but has some disadvantages. For example, real stars have absorption features, and so the definition of isophotal wavelength has to include interpolating over them to get an equivalent continuum. Figure 5.5 shows an example; the L dwarf spectrum is dominated by strong features just at the mean wavelength. If the interpolation is done in different ways, one can get different isophotal wavelengths for the same measurement on the same star! Furthermore, if one does not want every source to have its own characteristic measurement wavelength, flux-based bandpass corrections need to be used anyway.

In general, Figure 5.5 emphasizes that stellar photometry – where colors are compared without asking questions about the details – buries a lot of complications that need to be taken into account for physical photometry.

5.4.5 AB magnitudes
Another convention is “AB magnitudes”. These take the zero magnitude flux density at V and compute magnitudes at all bands relative to that flux density. Thus, they are a form of logarithmic flux density scale, with a scaling factor of $-2.5$ and a zero point of $\sim 3630$ Jy.
They have the virtue that conventional corrections, such as for interstellar extinction (tabulated in magnitudes), can be applied directly to them. Be careful, though, because away from the visible range AB magnitudes can differ substantially from Vega ones, e.g., \( m_K^{(AB)} - m_K^{(Johnson)} \approx 2 \). Forgetting these differences can resulted in serious mistakes.

5.4.6 Calibration in other spectral ranges

Other spectral ranges lack celestial calibrators that are basically modified blackbody radiators and that can be modeled and understood to the necessary level for accurate calibration from first principles. Thus, they rely more on direct measurements with specialized equipment to establish an absolute calibration. For example, in the radio, the primary standards are measured with horn antennas, which have cleaner beams and are easier to model than a paraboloid with a feed antenna (as discussed in Chapter 8). The calibrators are then tied in with other standards with conventional radio telescopes. The accuracies are \( \sim 10 - 15\% \) but can be improved to \( \sim 5\% \) with special procedures (Maddalena & Johnson 2005).

In the X-ray, the primary calibration is obtained on the ground prior to launching a telescope. In practical arrangements, the calibrating source is not quite at infinite distance, so the illumination is not exactly parallel, and there is a resulting uncertainty in telescope throughput. The calibration can be checked using celestial sources, but the results are very dependent on models for those sources. The estimated uncertainties for ACIS on Chandra are 5% for 2 – 7kev and 10% for 0.5 – 2 kev (Bautz 2000 and references therein). X-ray imaging is usually conducted in very wide spectral bands, so bandpass corrections must be calculated by convolving the response with the estimated spectra of the sources. The quoted X-ray fluxes are almost always linked immediately to the assumed source spectrum.

5.4.7 An Example

Suppose you got the following measurements of two stars (ADU means analog-to-digital unit and is the output of your digitizer):

<table>
<thead>
<tr>
<th>Band</th>
<th>J(1.25μm)</th>
<th>K(2.20μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star Y</td>
<td>25 ADU</td>
<td>1000 ADU</td>
</tr>
<tr>
<td>Star Z</td>
<td>200 ADU</td>
<td>110 ADU</td>
</tr>
<tr>
<td>sky</td>
<td>10 ADU</td>
<td>25 ADU</td>
</tr>
</tbody>
</table>

Assume that they were measured in different fields and that Star Z is a standard star with \( m_J = m_K = 12.00 \). All of your measurements used aperture photometry and the sky signal was measured with the same aperture as the stars. Star Y was measured at 1.1 air masses and Star Z at 1.3; the corrections for both bands are 0.2 magnitudes per airmass. Take the J filter to have a square transmission function from 1.15 to 1.35 μm and K to have a square transmission function between 1.95 and 2.45μm.

Compute \( m_J \) and \( m_K \) for star Y.

Compute the flux densities corresponding to the \( m_J \)'s and \( m_K \)'s. You can take:

\[
m_J = 0 \text{ at } 1603 \text{ Jy} = 1.603 \times 10^{-23} \text{ W m}^{-2} \text{ Hz}^{-1} \\
m_K = 0 \text{ at } 667 \text{ Jy} = 6.67 \times 10^{-24} \text{ W m}^{-2} \text{ Hz}^{-1}
\]

1. We compute sky- and air mass- corrected signals. The correction to unity air mass for Star Y is \( 10^{(0.4 \times 2.1)} = 1.0186 \), while it is 1.0568 for Star Z.

<table>
<thead>
<tr>
<th>Band</th>
<th>J(1.25μm)</th>
<th>K(2.22μm)</th>
</tr>
</thead>
</table>

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Photometry

<table>
<thead>
<tr>
<th>Star</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0186(25 – 10) ADU = 15.28 ADU</td>
<td>1.0186(1000-25) ADU = 993.1 ADU</td>
</tr>
<tr>
<td></td>
<td>1.0568(200-10) ADU = 200.8 ADU</td>
<td>1.0568(110-25) ADU = 89.8 ADU</td>
</tr>
</tbody>
</table>

2. Assume the output ADUs are linearly related to the detected flux density:

\[ \text{ADU}_X = C F_X \]

The reduction program assumes that zero magnitude gives 10,000 ADU in both bands. Then the instrumental magnitudes are

\[ m_{\text{inst}} = -2.5 \log (\text{ADU}/10000) = -2.5 \log (\text{ADU}) + 10 \]

or, in our case,

\[ m_{\text{inst}}(Y,J) = -2.5 \log (15.28) + 10 = 7.04 \text{ mag} \]
\[ m_{\text{inst}}(Y,K) = 2.51 \text{ mag} \]
\[ m_{\text{inst}}(Z,J) = 4.24 \text{ mag} \]
\[ m_{\text{inst}}(Z,K) = 5.12 \text{ mag} \]

From the results for Star Z, the correction to true magnitudes is

\[ 12 – 4.24 = 7.76 \text{ magnitudes at } J \]
\[ 6.88 \text{ magnitudes at } K. \]

Therefore, for Star Y, \( m_J = 14.80 \text{ mag} \), \( m_K = 9.39 \text{ mag} \), and \( m_J – m_K = 5.41 \text{ mag} \).

3. To compute flux densities, start with \( m_J \). By definition, the J flux density is fainter than that of a 0 magnitude star by

\[ 10^{14.80/2.5} = 8.32 \times 10^5 \]

Thus, its nominal flux density is 1.93 mJy

Similarly, star Y has a nominal flux density of 117 mJy at K.

Star Z has a flux density of 25.4 mJy at J and of 10.57 mJy at K.

4. However, the two stars have very different spectral slopes, so there may be a significant bandpass correction to get the best possible flux density estimates. We have little information about the slopes across the filter bands, so we need to guess. Suppose we fit a power law to the spectrum of star Y:

\[ S_Y(\nu) = C_Y \nu^\alpha \]

Then we can determine the index, \( \alpha \), as

\[ \alpha = \frac{\log (S(\nu_2)/S(\nu_1))}{\log (\nu_2/\nu_1)} = \frac{1.7826}{-0.2494} = -7.146 \]

Whereas Star Z can be taken from its J-K color of zero to be nearly Rayleigh-Jeans,

\[ S_Z(\nu) = C_Z \nu^2 \]

We now determine \( C_Y \) and \( C_Z \) by requiring the spectra to equal the nominal flux densities at the assigned wavelength for J, 1.25\( \mu \)m, equivalent to 2.40 \( \times 10^{14} \) Hz: \( C_Y = 1.1294 \times 10^{100} \) and \( C_Z = 4.4097 \times 10^{-31} \). We can simulate the measurement at J by convolving these spectra with the J filter passband and integrating. The results for the two stars should be identical if our nominal flux densities are correct.

\[ \text{Star Y signal} = 1.1294 \times 10^{100} \int_{2.222 \times 10^{14}}^{2.609 \times 10^{14}} \nu^{-7.146} d\nu = 7.69 \times 10^{10} \]
\[ \text{Star Z signal} = 4.4097 \times 10^{-31} \int_{2.222 \times 10^{14}}^{2.609 \times 10^{14}} \nu^2 d\nu = 9.9645 \times 10^{11} \]

Therefore, we predict a ratio of signals, Star Y/Star Z, of 0.0772, whereas we got 15.28/200.8 = 0.076096.
We conclude that the differing shapes of the spectra in the filter bandpass have biased our assignment of the flux density of star Y relative to that of Star Z slightly. To put them on the same basis, we should multiply the derived flux density of star Y by $0.0761/0.0772 = 0.986$, while retaining the same effective wavelength. That way, the integrals will correspond to the observed ratio.

Although the bandpass correction in this example is small, the size of the corrections grows roughly as the square of the fractional bandwidth, $\Delta \lambda / \lambda$ (and is a strong function of the spectra of the two sources being compared). In the X-ray, $\Delta \lambda / \lambda$ can approach unity and measurements are generally quoted in terms of a specific spectral shape assumed in the reduction. The mid-infrared N and Q bands (10.6 and 21 μm) are also quite broad, $\Delta \lambda / \lambda \sim 0.5$, and may need significant correction.

### 5.5 Other Types of Photometry

#### 5.5.1 Differential Photometry

Photometry can usually be made more accurate by using local references, either reference stars nearby on the sky to a science target, or comparing a spectral region of a source with a nearby one. ‘Spatial differential’ is the usual approach to monitoring a source for variability by observing nearby stars simultaneously with the measurement target. This gain comes ‘for free’ with imagers. Atmospheric effects, filter leaks, and time variability of the system can all be made to cancel to first order.

‘Spectral differential’ is useful when narrow spectral features can be measured by comparing signals from a narrow and broad filter, both centered on the feature (there are other strategies too). An example is a narrow filter centered on H$\beta$ and a broader continuum filter centered at the same wavelength, used for a quick type classification of hot stars based on the equivalent width of the H$\beta$ absorption. The ratio of the signals from these two filters can be calibrated on stars with spectroscopically measured equivalent widths and can then be used as a measure of the equivalent width on other stars.

#### 5.5.2 High speed photometry

There are a number of types of astronomical source that vary more quickly than can be measured with standard procedures. With arrays, the readout time is fixed by the speed of the transfer of signals (and the resulting charge transfer efficiency for a CCD), plus the speed of the output amplifier and the receiving electronics. However, with nearly all arrays, one can just get signals from a subsection, which can be read out at the optimal speed per pixel. If only a small fraction of the pixels in the array are being read out, then this subsection can be read much faster than the entire array while still maintaining the good performance (for the subsection only, of course). For example, a CCD can have the clocks run fast to shift the charge out over the "unwanted" area, dumping it by resetting. When you get to the subarray, the clocking is slowed down and the pixels are read out at the standard rate. When you get past the subarray, it is usually desirable to clock through the entire array so charge does not build up and bleed into regions where you do not want it. In theory infrared arrays allow full random access - you could address just the subarray. However, since usually shift registers are built in to shift the entire array out serially, usually one has to use a fast-advance strategy similar to that with the CCD to get to a subarray, and a similar readout strategy also. A few arrays have extra logic to allow direct addressing of a subarray (e.g., the Teledyne arrays for NIRCam). One can also use a custom-manufactured small array if one wants fast readout without the advancing over pixels.
Photometry

Higher speeds can be obtained by single-photon-counting using detectors with gain (e.g., photomultiplier, avalanche photodiode, solid state photomultiplier). Such devices are discussed in Chapter 3. The detectors used in the X-ray (pulse counting CCDs or microcalorimeters) and radio (heterodyne receivers) are intrinsically fast in response and can be adapted readily to high speed photometry.

5.5.3. High accuracy
Very high photometric repeatability has come to the fore in the past decade, for observing both the transits of planets (Figure 5.6) and also the oscillatory modes of stars (a field called astroseismology). An array detector is virtually required because an aperture photometer would modulate the signal with tiny telescope motions. It is possible to mitigate the pixel non-uniformity by putting many pixels across the point spread function. Accurate placement of the star on the array and holding it at the same position also improves the relative photometry, since it prevents many of the noise sources due to intrapixel sensitivity variations and similar causes. Analysis of the images to minimize systematic effects is critical. One effective technique is to convolve a template image of the field with the seeing at the time of observation and then to scale the template to match the image and to subtract the two. The photometry can then be performed on the residual image (Alard 2000). Other methods have also been used – simple aperture photometry image extraction and photometry works reasonably well -- and further progress is to be expected given that the success of transit photometry is dictated by the quality of the reductions.

Space telescopes are even better for transits and astroseismology. Although the many-pixel rule cannot be imposed, high quality data are being obtained by careful modeling of the results, taking advantage of the steady pointing and lack of effects from the atmosphere. HST and Spitzer have turned out to be extremely powerful in studying transits. MOST, CoROT and Kepler are three dedicated satellites for high accuracy photometry.

5.5. Polarization
5.5.1 Background
Electromagnetic radiation consists of transverse vibrations of the electromagnetic field. If they are incoherent, such vibrations can separately propagate through space without change and without interfering with each other. If the phase and amplitude of these separately propagating vibrations have no fixed relationship to one another, it is said that the radiation is unpolarized. If there are lasting relationships in phase and amplitude, then the light is at least partially polarized.

Processes that lead to significant polarization include:
- Reflection from solid surfaces, e.g., moon, terrestrial planets, asteroids
- Scattering of light by small dust grains, e.g., interstellar polarization
- Scattering by molecules, e.g., in the atmospheres of the planets
- Scattering by free electrons, e.g., envelopes of early-type stars
- Zeeman effect, e.g., in radio-frequency HI and molecular emission lines
- Strongly magnetized plasma, e.g., white dwarfs
- Synchrotron emission, e.g., supernova remnants, AGN

Light is polarized when the electric vectors of the photons in a beam of light are not random in direction. Two cases are generally distinguished: 1.) linear polarization, when the vectors are all parallel to each other and constant in direction; and 2.) circular polarization, where the vectors rotate at a constant rate and at the frequency of the light. In fact, these examples are the two limiting cases. The general case is elliptical polarization, where the electric vector rotates at the frequency of the light but its amplitude varies at two times the frequency. In this case, the tip of the electric field vector traces an ellipse on a plane that is perpendicular to the wave propagation direction (see Figure 5.7).

The four Stokes parameters are used to describe this behavior. They are total intensity $I$, linear polarization given by $U$ and $Q$, and circular polarization $V$. The general case, elliptical polarization should be viewed as the combination of all four, with an intensity of

$$\mathcal{I} P_E = \sqrt{Q^2 + U^2 + V^2} \quad (5.14)$$

where $P_E$ is the degree of polarization ($< 1$). The Stokes parameters are defined in terms of the electric vector from equation (5.13) as:

$$I = \langle E_x^2 \rangle + \langle E_y^2 \rangle \quad (5.15a)$$

$$Q = \langle E_x^2 \rangle - \langle E_y^2 \rangle = 1P_E \cos 2\beta \cos 2\theta = IP \cos 2\theta \quad (5.15b)$$

$$U = \langle 2E_x E_y \cos \phi \rangle = 1P_E \cos 2\beta \sin 2\theta = IP \sin 2\theta \quad (5.15c)$$
Photometry

\[ V = \langle 2E_x E_y \sin \varphi \rangle = I P_L \sin 2\beta = I P_V \]  

(5.15d)

where \( P = P_L \cos 2\beta \)  

(5.16)

is the degree of linear polarization and \( P_V = P_L \sin 2\beta \)  

(5.17)

is the degree of circular polarization, positive for right-handed and negative for left-handed; that is, \( \beta \) defines the relative amounts of linear and circular polarization. Thus, for \( \beta = 0^\circ \) the ellipse traced by the electric vector collapses into a line at angle \( \theta \), while \( \beta = 45^\circ \) yields a circle. The factor of two before \( \theta \) arises because a polarization ellipse is degenerate for rotations of 180\(^\circ\), while the two before \( \beta \) reflects a similar degeneracy for a 90\(^\circ\) rotation of an ellipse accompanied by swapping its major and minor axes. For example, \( \theta = 0^\circ \) is equivalent to \( \theta = 180^\circ \). In addition, \( \varphi = \varphi_x - \varphi_y \) is the phase difference between the \( x \) and \( y \) vibrations.

As applied in astronomy, the angles and intensity determine a polarization vector in spherical coordinates. \( \theta \) is defined as the angle between the long axis of the polarization ellipse and the direction toward the north celestial pole (NCP), measured counter-clockwise from the direction toward the NCP. \( \theta \) is called the position angle of the polarization vector.

Conversely, if we can measure the Stokes parameters, we can solve for the polarization vector as

\[ I = \frac{\sqrt{Q^2 + U^2}}{I} \]  

(5.18a)

\[ P = \frac{\sqrt{Q^2 + U^2}}{I} \]  

(5.18b)

\[ 2\theta = \arctan \left( \frac{U}{Q} \right) \]  

(5.18c)

The Stokes parameters are a convenient way to describe polarization because, for incoherent light, the Stokes parameters of a combination of several beams of light are the sums of the respective Stokes parameters for each beam.

Now imagine that we have a polarization analyzer, a device that subdivides the incident light in half, with one beam linearly polarized in a direction we define as the principal plane of the analyzer and the other beam polarized in the orthogonal plane. We designate the angle between the north celestial pole and the principal plane as \( \phi \). When light characterized by the Stokes parameters \( I, Q, U, \) and \( V \) falls on the analyzer, the intensities of the two beams emerging from it are (Serkowski 1962)

\[ I_{PP} = \left( \frac{T_I + T_R}{2} \right)^{1/2} \left( I + \left( \frac{T_I - T_R}{2} \right)^{1/2} \right) (Q \cos 2\phi + U \sin 2\phi) \]  

(5.19a)

\[ I_{OP} = \left( \frac{T_I + T_R}{2} \right)^{1/2} \left( I - \left( \frac{T_I - T_R}{2} \right)^{1/2} \right) (Q \cos 2\phi + U \sin 2\phi) \]  

(5.19b)

where \( T_I \) is the transmittance of unpolarized light (e.g., measured with two identical analyzers oriented with their principal planes parallel) and \( T_R \) is the transmittance of two analyzers with their principal planes perpendicular. For simplicity, assume a perfect analyzer, \( T_I = 0.5 \) and \( T_R = 0 \). Then

\[ I_{PP} = \frac{1}{2} (I + Q \cos 2\phi + U \sin 2\phi) \]  

(5.20a)

\[ I_{OP} = \frac{1}{2} (I - Q \cos 2\phi - U \sin 2\phi) \]  

(5.20b)

Thus, measuring the two beams emergent from the analyzer at a number of angles relative to the direction toward the north celestial pole, \( \phi \), gives a set of values of \( I_{PP} \) and \( I_{OP} \) that can be solved for \( Q \)
Photometry

and \( U, I \) can be measured as just the total flux from the source. Circular polarization is conventionally measured by inserting into the beam ahead of the analyzer an optical component that converts circular to linear polarization and then measuring the strength of this linear polarization.

5.5.2 Optical elements

5.5.2.1 Analyzers

Conceptually, perhaps the simplest form of analyzer is a fine grid of parallel conductors, placed perpendicular to the beam of light. If the spacing between wires is five times their diameters, then the grid acts as an efficient analyzer for wavelengths longer than about five times the grid spacing. Waves with electric fields perpendicular to the grid lines cannot transfer much energy to the grid because the grid diameter is so small compared with the wavelength, so they are able to travel through the grid. However, if the electric fields are parallel to the grid, the photons are reflected (or absorbed) efficiently. Given current capabilities to produce fine grids of conductors by photo-lithography, such devices work well throughout the infrared and can be made to work reasonably well in the visible. A similar concept explains the operation of sheet polarizing material, used in the visible. Sheet analyzers can be fabricated in polyvinyl alcohol plastic doped with iodine. The sheet is stretched during its manufacturing so the molecular chains are aligned, and these chains are rendered conductive by electrons freed from the iodine dopant.

Another class of analyzer is based on the birefringence of certain crystals – that is, in certain orientations these crystals have different indices of refraction for light polarized in different directions. Examples include magnesium fluoride, calcite, sapphire, and lithium niobate. A simple analyzer can be made with a plane-parallel calcite plate in front of the telescope focal plane, producing images in orthogonal polarization directions separated by 0.109 times the thickness of the plate (at 0.55 \( \mu \)m). However, because of the differing indices of refraction, the two images come to foci at different distances behind the plate. This issue can be avoided by using two (or more) plates of material. A well-known example is the Wollaston prism (Figure 5.8). It consists of two triangular right prisms cemented (or contacted) along their bases and with their polarization axes oriented orthogonally. When it encounters the interface between the prism, a beam of light finds that one polarization direction is transitioning from a medium with a relatively high refractive index to one with a relatively low one, and the other direction is transitioning from low to high. The result is that the two polarizations are deflected in opposite directions. Wollaston prisms can divide the two polarizations by 15 to about 45° in this fashion. There are a large number of alternative arrangements that perform functions similar to that of the Wollaston prism, e.g., the Glan-Thompson prism transmits one polarization undeviated but absorbs the other.

5.5.2.2 Retarders/wave plates

If one direction of polarization is retarded in phase by 180° relative to the other, the direction of the polarization is rotated, as shown in Figure 5.9 (see also equations 5.15 and 5.18). A simple retarder can be made of a plane-parallel plate of birefringent material, suitably cut and oriented. The specific case of
a 180° phase shift is called a half-wave-plate. In general more complex realizations of half-wave-plates use multiple materials to compensate for color differences and give a relatively achromatic phase shift.

By symmetry, a half-wave-plate with its optical axis oriented in the direction of polarization does not rotate the plane of polarization of the wave. If the optical axis is 45° from the plane of polarization, the half-wave-plate rotates the polarization 90° (see Figure 5.9), if the optical axis is 90° off the plane the rotation is by 180° (i.e., by symmetry, we are back to the beginning – no effective rotation).

Another special case applies when there is a 90° phase shift – then a perfectly plane polarized wave becomes perfectly circularly polarized (see Figures 5.9 and 5.10, and equation 5.15). Thus, a quarter-wave-plate converts linear polarized light to circular polarization, or circular to linear.
Photometry

There are other types of retarder besides passive plates of birefringent materials. One example is the Pockels cell. This device is based on materials in which the birefringence can be altered by an electric field, such as lithium niobate or ammonium dihydrate phosphate (AD*P). Birefringence is also induced in normal optical materials by mechanical strain. This effect is used in photoelastic retarders, which vibrate a piece of material at its natural resonant frequency to provide rapidly periodically varying retardation (e.g., at 20 kHz).

5.5.3 Polarimeters

A very simple polarimeter could be based on placing an analyzer in the beam and making measurements at different rotation angles of the analyzer. However, such an arrangement would have limited accuracy. A fundamental problem is that the measured effect appears synchronously with the rotation of the analyzer, making the instrument susceptible to false indications of a polarized input. One example is wandering of the beam due to optical imperfections in the analyzer. In addition, the optical train behind it may transmit different polarizations with different efficiencies – for example, reflections off tilted mirrors induce polarization in the beam. Although it is possible to calibrate such false signals and remove them to some degree from the measured values, it is much better to use a polarimeter design that is less susceptible to them.

One such design is to rotate the entire instrument, rather than just the analyzer. In that case, the path through the instrument remains identical for the different angles and the effects of polarization-sensitive optics should not matter. However, the instrument cannot generally be rotated very fast; any changes in conditions (e.g., seeing or atmospheric transmission) can have a large influence on the measurements made at different times. Still, rotating the instrument can be a useful technique to probe possible instrumental errors.

More adaptable designs are based on placing a waveplate or other retarder in the beam before it encounters any tilted mirrors that might induce instrumental polarization. For a normal Cassegrain or Gregorian telescope, the two first reflections (primary and secondary) are nearly normal to the mirror surfaces and thus, by symmetry, also induce very little polarization if the retarder is placed directly in the beam formed by these two mirrors. An analyzer is placed behind the retarder, and can in fact be

![Figure 5.10. (a) two electric waves π/2 out of phase and (b) their superposition produces a circularly polarized wave, from http://webpages.ursinus.edu/lriley/courses/p212/lectures/node25.html](http://webpages.ursinus.edu/lriley/courses/p212/lectures/node25.html)
behind any number of tilted mirrors or other polarizing optical elements without significantly compromising the performance. In fact, any polarimetric activity by these other optical elements combines optically with the activity of the analyzer. For example, if the analyzer is immediately behind the retarder, fully polarized light will pass through the rest of the instrument and a polarizing reflection might result in some of this light being removed, but will not change the net modulation of the signal as the retarder is used to rotate the angle of polarization because the loss of efficiency will be in equal proportion for all signals.

Suppose that the polarization measurement is obtained by measuring the signals at different rotation angles of a half-wave-plate. The rotation of the angle of polarization will be $2\theta$ for a rotation of the wave plate by $\theta$, that is, changes in signal due to optical imperfections in the wave plate will have a different rotational dependence from changes due to a polarized input beam. These two improvements: 1.) immunity to polarization effects and other flaws in the instrument optics; and 2.) separating the signal from one-to-one correspondence with the motion of the analyzer, together yield a big gain in the accuracy with which polarization can be measured. A further gain arises because the wave plate can be

Figure 5.11. A spectropolarimeter (SPOL)
moved to a new angle quickly, so different measurements can be obtained over time intervals short compared with changes in atmospheric transmission, seeing, and other variable effects. In this manner high quality polarimetry can be conducted with a variety of instrumentation, including cases with strongly polarizing optical elements such as a diffraction grating (Figure 5.11).

Alternative types of retarder are based on Pockels cells or photoelastic devices. With them, there is no bulk mechanical motion associated with the rotation of the plane of polarization and in theory the polarization artifacts associated with the rotation of a conventional wave plate are avoided.

5.5.4 Measurement interpretation and error analysis

From equation 5.20, if we make a measurement of the polarization in the principal plane and in the orthogonal plane of our analyzer when it is at angle $\phi$ relative to the NCP, then

$$R = \frac{I_{pp} - I_{qp}}{I_{pp} + I_{qp}} = \frac{Q \cos 2\phi + U \sin 2\phi}{I} \quad (5.21)$$

Although this expression assumes a perfect analyzer, it can be generalized to a more realistic case in a straightforward way. For $\phi=0$, we get $R_0 = Q/I = q$, while for $\phi=45^\circ$, we get $R_{45} = U/I = u$. We can adapt equations (5.18) for the amount and angle of polarization.

$$P = \sqrt{q^2 + u^2} \quad (5.22a)$$

$$\theta = \frac{1}{2} \arctan \left( \frac{u}{q} \right) \quad (5.22b)$$

It is convenient to use a diagram of $q$ vs. $u$, with angles in $2\theta$, to represent polarization measurements (Figure 5.12). For example, different measurements can be combined vectorially on this diagram. To see how this works, suppose we made two measurements of a source, the first giving 2% polarization at an angle of $20^\circ$ and the second 2% polarization at an angle of $110^\circ$. These measurements are consistent with the source being unpolarized. If we put them on the $q – u$ diagram and combine them vectorially, we indeed get a result of zero.

{footnote: Linear algebra is often used to analyze the behavior of a series of polarimetrically-active optical elements, each of which is described by a “Mueller matrix.” The basics of polarimetry can be explained without this formalism, but anyone planning to get more familiar with the field should expect to encounter it.}

Error analysis for polarimetry is generally straightforward, except when it comes to the position angle for measurements at low signal to noise. Assume that the standard deviations of $q$, $u$, and $P$ are all about the same. Then the uncertainty in the polarization angle is

$$\sigma(\theta) \approx 28.65^\circ \left( \frac{\sigma(P)}{P} \right) \quad (5.23)$$
Thus, nominally a measurement at only one standard deviation level of significance (that is, a non-detection) achieves a polarization measurement within 28.65°. This high accuracy is non-physical – the probability distribution for θ at low signal to noise does not have the Gaussian distribution assumed in most error analyses (e.g., Wardle and Kronberg 1974). Similarly, P is always positive and hence does not have the Gaussian distribution around zero assumed in normal error analysis.

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Further Reading
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