Centimeter-wave Radio Astronomy
Much of this material is from Michael Garrett
Instead of optics and mixers, the electric fields of the photons are picked up with antennae and conveyed to the mixers. Dipoles are the simplest:

Types of antenna or antenna arrays:

Wavelength $> \sim 1$ metre: simple wire antennas can be used...

**Dipole**

$\text{Length} \leq \lambda$

**half-wave dipole**

$\text{Length} \leq \lambda/2$

E-field of incoming radiation sets up currents in the antenna $\Rightarrow$ voltages can be measured across a resistor. Dipole length must be kept short, $\text{Length} \leq \lambda$. N.B. any antenna is only sensitive to one polarisation (current is induced by field that is parallel to dipole length).

The simplest antenna is the half-wave dipole $\text{Length} \sim \lambda/2$. The Directivity (max gain) of a half-wave dipole is measured in dBi, dB above an isotropic (non-directional) radiator. For a dipole $G \sim 1.6$ dBi (not much better than isotropic!). The gain can be improved by combining together the output of several dipoles arranged in an array.
Dipole arrays have increasingly directional response with more elements. We can use such a device to collect the energy from a parabolic reflector radio telescope (the reflector version at the bottom). The response pattern at the paraboloid is described as the illumination pattern (since it is what would be illuminated if the telescope were being used to transmit rather than receive). The dipole array has additional response zones that produce sidelobes.

Figure 1.2.6. Polar diagram for a four-element collinear array with a mesh reflector.
At short cm (and shorter) wavelengths, a feedhorn is the preferred way to illuminate the telescope primary.

The feedhorn’s interior is corrugated in order to increase the surface impedance, so that the wave does not set up voltages in the surface material, but is channelled into a dipole at the end of the horn.

The feedhorn (or “feed”) is designed to evenly illuminate the antenna surface. The angle subtended by the reflector as seen by the feed strongly influences the types of feeds which may be used, and the details of their design.

Corrugated horns (above) are the most common.
If the illumination is uniform, the antenna response is just the familiar Airy pattern.

The response of a uniformly illuminated circular parabolic antenna of 25-metre diameter, at a frequency of 1 GHz.
However, there are a lot of other options for primary mirror illumination.

The illumination of the antenna surface by the feed is usually not uniform.

Feeds are usually designed to under illuminate edges of the dish - in order to avoid spillover from the ground.

Such a design produces a larger beam but smaller side-lobes. Cases (b), (c), (d), (e) - right.

Over illumination of the edges results in a narrower beam (better resolution) but high sidelobes. Cases (f) and (g) - right.
Antenna surface efficiency

According to the Ruze (1966) formula, the surface efficiency of a paraboloid is well described by:

$$\eta_{sf} = e^{-\frac{(4\pi\sigma/\lambda)^2}{2}}$$

where sigma is the r.m.s. error in the surface of the antenna.

Or re-arranging:

$$\frac{\sigma}{\lambda} = \frac{1}{4\pi} \sqrt{-\ln(\eta_{sf})}$$

e.g. For a surface efficiency of 0.7 (typical target value), the required surface error (r.m.s.) is $\sim \lambda/20$.

$$\Rightarrow$$ at 7 mm (43 GHz) the surface accuracy must be $\sim 350$ micron.

$$\Rightarrow$$ many different forces acting on an antenna and its surface...
Antenna Performance (see Napier SIRA)

The antenna aperture efficiency $\eta = \frac{\text{Power collected by feed}}{\text{Power incident on antenna}}$

There are many different potential loss factors: $\eta = \eta_{sf} \eta_{bl} \eta_{sp} \eta_{t} \eta_{misc}$ [6]

$\eta \sim 0.4 \iff$

- Surface efficiency $\sim 0.8$
- Aperture blockage efficiency $\sim 0.8$
- Feed spillover efficiency $\sim 0.8$
- Feed illumination efficiency $\sim 0.8$
- Misc. - other minor losses e.g. feed mismatch.

Feed does not illuminate all of antenna surface equally
Radio telescopes/astronomers often measure the (Spectral) “Flux Density” of a source.

The Flux density \( S \), is the power received \( P \) within a certain frequency band \( dv \), via a certain effective area \( A \) with efficiency \( \eta \):

\[
S = 2 \frac{P}{\eta A dv} \quad \text{(Watts m}^{-2} \text{ Hz}^{-1}) \quad [3]
\]

The factor of 2 above is because the measurement of power is traditionally only made via one polarisation channel, and it is assumed that the other hand will contribute the same amount of power.

The unit of Flux density is the Jansky (Jy): \( 1 \text{ Jy} = 10^{-26} \text{ Watts m}^{-2} \text{ Hz}^{-1} \). Typical units include millijansky (mJy) and microjy (uJy). Nanojy requires SKA!

E.g. sources as bright as 1 Jy are relatively rare. For a Westerbork antenna \( D \sim 25 \text{ metres} \); efficiency \( \sim 0.7 \) (\( \Rightarrow \) effective Area \( \sim 340 \text{ sq metres} \)); observing bandwidth 20 MHz. A source of 1 Jy produces a signal of only \( \sim 7E-17 \) Watts!

Since the Flux density is proportional to the distance, \( d^2 \), between the observer-source, it is not an intrinsic property of the source i.e. it does not reflect the real source luminosity e.g. quiet sun has a flux density of 1000 Jy at 6cm,
An isotropic antenna is a (mythical) antenna that collects (or radiates) energy uniformly in all directions. The gain of an antenna is defined as $G$,

$$ G = \frac{\text{Power radiated in a specific direction}}{\text{Power radiated in that direction by an isotropic radiator}} $$

Note that the average gain of any antenna is always 1 if calculated over all angles. The gain of an antenna can also therefore be written as:

$$ G = \frac{\text{Solid angle subtended by a sphere}}{\text{Solid angle of antenna beam}} \quad \text{or} \quad G = \frac{4\pi}{\Omega_A} = \frac{4\pi D^2}{\lambda^2} \sim \frac{4\pi A_e}{\lambda^2} \quad [11] $$

N.B. $A_e$ is the effective area, and is always smaller than the geometric area (see eqn 6).

$A_e = \eta A$ where $\eta$ is typically in the range of 0.4 - 0.8.

Note that from [11] we can also write:

$$ A_e \sim \lambda^2/\Omega_A \quad [12] $$
Antenna Gain and Performance

The angular response of a parabolic antenna with aperture size, \( D \), observing at a wavelength lambda, is diffraction limited and focused into a cone of solid angle:

\[
\Omega_A = \frac{\pi}{4} \theta^2 = \frac{\pi \lambda^2}{4 D^2} \sim \frac{\lambda^2}{D^2} \quad [8]
\]

Note if we substitute this into eqn 6 we get:

\[
S' = \frac{2kT}{\lambda^2} \left( \frac{\lambda}{D} \right)^2 = 2kT/D^2 \quad [9]
\]

Noting that the power (P) into the receiver is given by eqn [3], by equating with [9] we can write:

\[
P = kTdv \quad T \text{ is known as the antenna temperature, usually denoted } T_A. \quad [10]
\]
Eqn[10] is equivalent to the power associated with a resistor placed in a thermal bath at a temperature $T$ - the so-called Johnson-Nyquist formula.

The electrons in the resistor undergo random thermal motion, and this random motion causes a current to flow in the resistor. On average there are as many electrons moving in one direction as in the opposite direction, and the average current is zero. The power in the resistor however depends on the square of the current and is not zero. The power is well approximated by the Nyquist formula:

$$P = kTdv$$

where $k$ is the same Boltzmann constant as in the Planck law.

In analogy with this, if a power $P$ is available at an antenna's terminals the antenna is defined to have an antenna temperature of

$$T_A = P/(kdv)$$

Note that $T_A$ is not the physical temperature of the antenna!
e.g. A 25-metre telescope, observing a 100 millijansky (mJy) radio source measures an antenna temperature, $T_a$, of 0.023 Kelvin!

N.B. for a source that is unresolved the antenna temperature, $T_a \ll T_b$ (the source brightness temperature). In fact:

$$T_a \sim T_b \times \text{beam filling factor}$$

The beam filling factor is the fraction of the telescope beam occupied by the source (see right). The radio cores of AGN are usually $\ll$ the antenna beam - filling factors in excess of $\sim 1 \times 10^{-12}$ are then typical.

To measure the brightness temperature of very small sources, like AGN, Very Long Baseline Interferometry must be used.

Note also from equation [9] that for an unresolved source, the measured signal will increase as the diameter of the telescope increases.
Appreciating the scale of large radio telescopes....

How can we possibly achieve a 350 μm accuracy (the thickness of three human hairs) – over a 100 metre diameter surface – an area equal to 2 football fields!

==> “active surface”.
GBT Surface has 2004 panels average panel rms: 68µm.

More than 2000 precision actuators are located under each set of surface panel corners.

Actuator Control Room (left): 260000 control and supply wires terminate in this room!
How big can parabolic radio telescopes be?

As the size (diameter) of a radio telescope increases, the gravitational and wind loads on the structure become difficult to manage. The worst problem is the problem of surviving a gale-force wind. The degree of wind distortion between paraboloids of different diameters ($D$) scales as $D^3$.

The cost of antennas also scales roughly as $D^3$.

Telescopes like the Jodrell Bank Mark V (right) with a diameter of ~ 305 metres (1970), will probably always remain in model form!
The ability to sense the photon phase allows radio telescopes to take very “different” forms, particularly at low frequencies/long wavelengths. As a result, large area telescopes can be built at low cost (and without defying gravity).

A Specialized Solution

Gauribidanur Telescope (India)

Operates at ~ 10 meters wavelength. The beam is steered by adjusting the relative phases of the outputs of the individual antennae.
Interferometers provide a more general solution that can be applied at high frequencies (including into the submm).

Here is Westerbork, a linear E-W array of 14 antennae.
The VLA is a Y-shaped Array
With the use of coherent (heterodyne) receivers, it is no longer necessary to match geometric path lengths for an interferometer to work; the path differences can be removed electronically.

**Interferometric imaging**

A complex correlator computes the sine and cosine components simultaneously and introduces a compensating instrumental delay $\tau_i$ (see figure below) in one of the antennas which is a good (but imperfect) estimate of the true geometrical delay $\tau_g$ (the various geophysical effects are hard to predict).

The difference between the geometrical delay and the instrumental delay is the delay tracking error, $\tau$

$$\tau = \tau_g - \tau_i$$

In a modern interferometer the instrumental delay is introduced via a digital phase shift.
Let us consider a 2-element interferometer observing at frequency, \( \nu \), a radio source with a brightness distribution, \( I_\nu(s) \), then the output of a complex correlator is the power received per unit bandwidth, \( dv \), from an element of the radio source, \( ds \) is:

\[
r_{12} = A(s) I_\nu(s) e^{i2\pi\nu \tau} ds dv
\]

The total response is obtained by integrating over the solid angle subtended by the source:

\[
R(B) = \int \int A(s) I_\nu(s) e^{i2\pi\nu(\frac{1}{c}(B \cdot s) - \tau_i)} d\Omega dv \quad [9]
\]

For parabolic antennas \( A(s) \) is usually considered to be zero outside of the fwhm of the antenna primary beam. So in practice the integration is restricted to this area.
Just as in the infrared and optical, the visibility function is a key parameter.

For an extended source $\vec{s} = \vec{s}_o + \vec{\sigma}$ and noting that that $s_o$ and $\vec{\sigma}$ are essentially perpendicular to one another on the celestial sphere, we can write:

$$\vec{B}.(\vec{s}_o + \vec{\sigma}) = \vec{B}.\vec{s}_o + \vec{b}.\vec{\sigma}$$

Substituting this in eqn[9] we can write:  

$$R(\vec{B}) = V(\vec{B})A_0 e^{i2\pi\nu(\frac{1}{c}(\vec{B}.\vec{s}_o) - \tau_i)} d\nu$$

where:

$$V(\vec{B}) = \int \int A'(\vec{\sigma})I(\vec{\sigma}) e^{i2\pi(\frac{\vec{B}}{\lambda}.\vec{\sigma})} d\Omega \quad [10]$$

and $A'(\sigma) = A(\sigma)/A_0$

is the normalised beam pattern with $A_0$ being the response at the beam centre in the direction of $s_o$.

The integral above, $V(\vec{B})$ is the visibility function and from its form you can probably see already it is the Fourier Transform (FT) of the source brightness distribution. In principle, we may therefore recover the source brightness distribution by performing the inverse FT on the visibility function.
Multiple telescopes provide many baselines.

An interferometer with $N$ antennas contains $N(N-1)/2$ interferometer pairs.

The instantaneous synthesized beam projected on the sky (the point-source response obtained by averaging the outputs of all pairs - thin curves $r_{12}, r_{13}, r_{14}, r_{23}, r_{24}, r_{34}$) rapidly approaches a Gaussian (thick curves e.g. $r_{12} + r_{13} + r_{14} + r_{23} + r_{24} + r_{34}$) as $N$ increases.

The synthesized main beam of the four-element interferometer (left) is nearly Gaussian with angular resolution $\sim \lambda / b$.
Longer baselines (e.g. \( r_{12} \)) have a narrower angular fringe spacing projected on the sky. Their response is sensitive to very compact objects with scales \( \sim \) the fringe spacing.

Shorter baselines (e.g. \( r_{13} \)) have a larger angular fringe spacing projected on the sky. They are sensitive to extended objects on the same scale as the fringe spacing.

By measuring the response of each interferometer spacing, we can visualise how it's possible to build-up an image of the sky!
Aperture synthesis

As a telescope collects measurements of $V(u,v)$ we talk of “filling in” the uv-plane. In this pursuit, astronomers are greatly aided by the rotation of the Earth!

As the Earth rotates, the projected baseline vector, $B$ changes as seen by the source. In this way we collect many measurements of $V(u,v)$ - these measurements are called “visibilities”.

The visibilities are values for the amplitude and phase of the response of the correlator for each interferometer (baseline) at a given time (equivalent to a measurement in the uv-plane).

Consider a source located at the pole, and a simple 2 telescopes interferometer (see diagram above). As the Earth rotates, the source sees the baseline vector rotating too, tracing a circle in the sky. For a source at the pole, the projection of the baseline length does not change, but its orientation does. In the uv-plane, the baseline traces out a circle:
If we add more antennas, say in between the 2 pictured above (red dots) we begin to fill-in the uv-plane...

Mathematically we say that each visibility is a Fourier component of the sky brightness.

The better we fill-in the uv-plane, the higher the quality and fidelity of the images.

In the case of Westerbork we have 14 antennas, and we can also move the antennas on rails!
For Westerbork, a 6 x 12 hour observing run (moving some of the antennas after each 12 hr run) produces almost full uv-coverage (see above).
Uv-coverage: WSRT observing at 20cm for ~12 hours ("full-track").

For this high-declination source, excellent uv-coverage is obtained.
So, for sources not located at the pole, the projected baseline length appears to change as the Earth rotates. In particular, for sources at declinations $< 90$ deg, the uv-tracks become elliptical:

uv-coverage of the WSRT for various source declinations.

The uv-tracks are only plotted when the source is 10 degrees above the local horizon. Below 10 degrees the data are often discarded because they are partially corrupted by the atmosphere at low elevations.

The maximum value of $u$ equals the antenna separation in wavelengths. For an E-W array, the maximum value of $v$ is smaller by the projection factor: i.e. $v_{\text{max}} = u_{\text{max}} \cos(90 - \delta) = u_{\text{max}} \sin(\delta)$ where $\delta$ is the source declination. If the interferometer has more than two elements, or if the spacing of the two elements is changed via rail tracks, the (uv) coverage will become a number of concentric ellipses.

Note that the uv-coverage becomes poorer for low-declination sources and becomes 1-dimensional for source located at zero declination. It is very difficult for the WSRT to make good images of low-declination sources (dec $< 20$deg) even after a 12 hour observing run.
Full synthesis (sometime referred to as “full track”) VLA observations of sources at different declinations (c.f. WSRT previous slides):

Tracks: elevation > 10 degrees
uv-plot of the amplitude of the source visibility plotted as a function of increasing baseline length.

Note that the short baselines are sensitive to larger scale emission (c.f. to the short baselines) and see more flux density than the short baselines.

Note also this is an idealised data set - pre-edited and calibrated (courtesy of Raffaella Morganti).
Fringes projected on to the sky for a short VLA baseline
Fringes projected on to the sky for a long VLA baseline
Right: Amplitudes and phase plotted against time.

N.B. baseline length increases from top to bottom.

The visibility function beats faster as the baseline length increases.
Recall FT of the sampling function (uv-coverage) produces the Point Spread Function (PSF) usually referred to in radio astronomy as the DIRTY BEAM.

The imaged field is the “primary beam,” $\lambda/D$ for the single telescopes.
A slice through the dirty beam”

Main synthesised beam response has a width of $\sim \frac{\lambda}{B_{\text{max}}}$

Note the side-lobes for WSRT (at this declination) are around 2-3% of the main response - pretty good!
Dirty beam and dirty map (no cleaning):

Note the pattern of the dirty beam (left) is imposed on every discrete source in the dirty map.
We can write that the dirty image is the convolution of the true image with the dirty beam (sampling function):

\[ I^D(x,y) = I \ast B \]

where \( B \) is the dirty beam

The CLEAN algorithm attempts to deconvolve the dirty beam from the dirty map:

(i) Assume the sky is +ve and that sources can be represented by a collection of point source delta functions.

(ii) make the dirty map, and find the position and intensity of strongest source in map

(iii) subtract some scaled fraction (e.g. 10%) of the dirty beam response at this position from the dirty map and store the position and intensity as a “CLEAN component” of the dirty map.

(iv) find strongest source in the new dirty map, subtract some scaled fraction of the dirty beam once again and add this to the CLEAN component file and repeat the same process many times (e.g. 1000 iterations) until only noise is left in dirty map.

(v) The final clean map is produced by taking all the CLEAN components, adding back the residual noise left in the dirty image (see iv), and then convolve this image with a Guassian that was fitted to the central lobe of the dirty beam.
CLEAN maps with 50 clean components (cc), 100 cc, 1000 cc and 5000 cc.

After subtracting 5000 clean components most of the artefacts of the dirty beam (but not all) are gone. Errors persist around the very brightest sources in the field where the subtraction must be most accurate.
CLEAN maps and the residual maps with 50 clean components (cc), 100 cc, 1000 cc and 5000 cc.

After subtracting 5000 clean components the residual map left is mostly noise.
CLEAN maps and the residual map with 50 clean components (cc), 100 cc, 1000 cc and 5000 cc.

Bright sources out of the primary beam leave residuals that are very hard to remove well.

After subtracting 5000 clean components the residual map left is mostly noise.
For many complex sources, all VLA configurations (i.e. as many baseline lengths as available) are required in order to get a good "picture" or to properly understand the source:

Since the configurations of the VLA are separated by many months (even years), care must be taken in combining data from different configurations - potential problems include source variability and a stable calibration from one configuration to another.
Which one is the real M33?
Effelsberg filled aperture to the left, VLA below.
Which one is the real M33? Effelsberg filled aperture to the left, VLA below.

Answer: The filled aperture map has 3 – 4 times as much flux as the aperture synthesis image. The missing short baselines result in the diffuse, extended emission not being detected by the VLA.
Here’s what the galaxy really looks like: nonthermal to the left (steep spectrum) and thermal to the right (flat spectrum).
The future: interferometric arrays of many small telescopes

Current-generation interferometers have relatively few, large antennae. For example, the VLA has 27 antennae, each 25 meters in diameter. The result is that the primary beam, over which the VLA images, is relatively small ($\lambda/D$ for the antenna size, about 0.5 degree at 21 cm). With cheaper correlators – the electronics that link the antenna signals – it is possible to have more, smaller antennae. The Allen Telescope Array (top) is to have 350 6-meter telescopes, so 4 times the field of the VLA with about the same collecting area. The Square Kilometer Array (not funded) is planned to have thousands of antennae. The issue is that the number of correlators goes as $N(N-1)$, where $N$ is the number of antennae, so the SKA would be prohibitively expensive with today’s technology.