Astrometry
Coordinate Systems

• There are different kinds of coordinate systems used in astronomy. The common ones use a coordinate grid projected onto the celestial sphere. These coordinate systems are characterized by a fundamental circle, a secondary great circle, a zero point on the secondary circle, and one of the poles of this circle.

• Common Coordinate Systems Used in Astronomy
  – Horizon
  – Equatorial
  – Ecliptic
  – Galactic
The Celestial Sphere

A **great circle** is the intersection on the surface of a sphere of any plane passing through the center of the sphere. Any great circle intersecting the celestial poles is called an **hour circle**.
Horizon Coordinate system

**Zenith:** The point on the celestial sphere that lies vertically above an observer and is $90^\circ$ from all points on the horizon.

**Nadir:** The point on the celestial sphere that lies directly beneath an observer. It is diametrically opposite the zenith.

The **celestial meridian** is a great circle that intersects the zenith, the nadir, and the celestial poles.

The **astronomical horizon** is a great circle on the celestial sphere which is perpendicular to the zenith-nadir axis.
Horizon Coordinate System (alt-az system)

Fundamental circle: astronomical horizon
Zero point: north cardinal point

**Altitude (a):** is the angular distance north (+) or south (-) of the horizon. It is measured along a vertical circle through the celestial body. (0°: rising, 90°: zenith)

**Vertical circle:** great circle that is perpendicular to the horizon and intersects the zenith

**Azimuth (A):** of a body is its angular distance measured eastwards along the horizon from the north point* to the circle. (0° to 360°)

**Zenith Distance** is the compliment of the altitude (90 – a)

*North cardinal point: intersection of the celestial meridian and the horizon that is also closest to the north celestial pole.
Horizon Coordinate System

• Advantage: Easy to use system. It is often useful to know how high a star is above the horizon and in what direction it can be found. Many telescopes use the alt-az mounts because of lower cost and greater stability.

• Disadvantage: because any coordinates depend on:
  *place of observation
    (because the sky appears different from different points on Earth)
  *on the time of observation
    (because the Earth rotates, the zenith is always moving relative to the stars)

• We need a system of celestial coordinates that is fixed on the sky, independent of the observer's time and place. For this, we change the fundamental circle from the horizon to the celestial equator.
1st Equatorial System (Hour angle, Declination)

- The **celestial meridian** (observer’s meridian) is a great circle which intersects the zenith, the nadir, and the celestial poles.
- The **declination** ($\delta$) of $X$ is the angular distance from the celestial equator to $X$, measured from $-90^\circ$ at the SCP to $+90^\circ$ at the NCP.
- The **Hour Angle** or HA ($H$) of object $X$ is the angular distance between the meridian of $X$ and celestial meridian. It is measured westwards in hours, 0h-24h.
- This system is still dependent on the time of observation, but an object's declination generally doesn't change rapidly, and its Hour Angle can be determined quite simply, given the time and the location.
2nd Equatorial System (Right Ascension and Declination)

- Most widely used coordinate system (for objects outside the solar system).
- Fundamental circle: celestial equator.
- Zero point: vernal equinox.
- The **right ascension** of a point is the angular distance measured eastward along the celestial equator between vernal equinox and the hour circle intersecting the point. It is measured in hours, minutes, and seconds from 0 to 24 hours.
- The **declination** of a point is the angular distance above or below the celestial equator. It is measured in degrees, minutes, and seconds from $-90^\circ$ to $+90^\circ$ with $0^\circ$ being on the celestial equator. Negative degrees indicate that the point is south of the celestial equator, while positive degrees indicate that it is north of the celestial equator.
Transforming between coordinate systems

Three general methods to do this:

1. Spherical trigonometry (Textbook on Spherical Astronomy by W.M. Smart)
2. Using rotation matrices
3. Find a web page where this is all worked out
   a. Generate a vector in the original coordinate system
   b. Convert the vector to another coordinate system by rotating the coordinates using matrix multiplication
   c. Convert the vector to the angles of the new coordinate system
Transformation between Horizon system to Hour angle and declination

The transformation from hour angle (h) and declination (δ) to horizon system: azimuth (a), zenith distance (z)

I(a,z) = R₃(180°) R₂(90°− φ) I(h,δ),
where φ is the observer’s geographic latitude

\[
\sin z \cos a = \cos \phi \sin \delta - \sin \phi \cos \delta \cos h \\
\sin z \sin a = - \cos \delta \sin t \\
\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h
\]

The inverse relationship is:

\[
\cos \delta \cos h = \cos \phi \cos z - \sin \phi \sin z \cos a \\
\cos \delta \sin h = - \sin z \sin a \\
\Sin \delta = \sin \phi \cos z + \cos \phi \sin z \cos a
\]
Sidereal Time

Which stars are on your local meridian?

- It depends on the time at which you observe. In fact, it depends on both the date and the (clock) time, because the Earth is in orbit around the Sun.
- Consider the Earth at position E₁ on the diagram. The star shown is on the meridian at midnight by the clock. But three months later, when the Earth reaches position E₂, the same star is on the meridian at 6 pm by the clock.
- Our clocks are set to run (approximately) on solar time (sun time). But for astronomical observations, we need to use sidereal time (star time).

Consider the rotation of the Earth relative to the stars.

We define one rotation of Earth as one sidereal day, measured as the time between two successive meridian passages of the same star. Because of the Earth's orbital motion, this is a little shorter than a solar day. (In one year, the Earth rotates 365 times relative to the Sun, but 366 times relative to the stars. So the sidereal day is about 4 minutes shorter than the solar day.)
Hour Angle and Time

**Hour angle**: is the angle measured westwards along the celestial equator from the observer’s meridian to the hour circle of the celestial body.

Another way to think about it is: the *hour angle* of a celestial body: is the time elapsed since the celestial body’s last transit of the observer’s meridian.

If HA = -1 (it will be 1 hour before the object is on the meridian. This is the same as HA = 23.

If HA = 0 (it is on the meridian)

If HA = 2 (it has been 2 hours since the object was on the meridian.)
Local Sidereal Time and Right Ascension

The Local Sidereal Time (LST) = Hour Angle of the vernal equinox.

- Let's suppose that LST = 1h. This means that the vernal equinox has moved 15° (1h) west of the meridian, and now some other star X is on the meridian. But the Right Ascension of star X is the angular distance from the vernal equinox to X = 1h = LST. So at any instant, Local Sidereal Time = Right Ascension of whichever stars are on the meridian.
- And in general, the Hour Angle \( \text{Object} = \text{LST} - \text{RA}_{\text{Object}} \)
Ecliptic Coordinate System
(for completeness)

- When dealing with the positions and motions of solar system objects, it is often more convenient to refer positions to the mean orbital plane of the solar system using *ecliptic coordinates*.
- Fundamental circle is the ecliptic.
- Zero point: vernal equinox.
- **Ecliptic latitude**, $\beta$, is analogous to declination, but measures distance north or south of the ecliptic, attaining $+90^\circ$ at the *north ecliptic pole* (NEP) and $-90^\circ$ at the *south ecliptic pole* (SEP).
- **Ecliptic longitude**, $\lambda$, is analogous to right ascension and is measured from the vernal equinox, in the same direction as right ascension but along the ecliptic rather than the celestial equator.
Galactic Coordinate System

The equatorial system is geocentric and thus provides an inappropriate viewpoint for problems of Galactic structure and dynamics.

• The fundamental circle is galactic equator which is coincident with the plane of the Milky Way Galaxy (shown in red). The plane is inclined at an angle of 62.87 degrees to celestial equator.)

• Zero point lies in the direction of the galactic center as seen from Earth. The zero is about 0.07 degrees from true center.

• Galactic latitude (b) of a celestial body is its angular distance north (+) or south (-) of the galactic equator (it ranges from 0 to 90°)

• Galactic Longitude (l) of a celestial body is its angular distance (from 0 to 360°) from the nominal galactic center measured eastwards along the galactic equator to the intersection of the great circle passing through the body.
Changes of Celestial Coordinates

• The are several effects that cause the coordinates of a star to deviate from those given in star catalogs.
  – Precession
  – Nutation
  – Proper Motion
  – Parallax
  – Refraction
  – Aberration of light
Obliquity or axial tilt is the angle between the rotational axis and a line perpendicular to its orbital plane.
Earth’s Axial Precession

The Earth’s axis is not fixed in space, but like a spinning top, the direction of the rotation axis executes a slow precession with a period of 26,000 years.

Cause: the Moon and the Sun exert tidal forces on the equatorial bulge of the Earth, and there are small effects from the rest of the planets in the solar system. This complex set of forces causes the rotational axis to gyrate, or precess, about the orbital axis. -> North and South Celestial Poles to circle around the Ecliptic Poles.
Precession, Nutation & Polar Motion

This combined movement has short and long period components which are usually broken down into 'corrections' as:

- **luni-solar precession** (period about 26,000 years)
- **planetary precession** (shift of ecliptic due to orbits of planets)
- **nutation** (major period 18.6 years and amplitude 9.5 arc-seconds)
- **polar motion** (is the movement of the Earth’s rotation axis across its surface due to internal dynamics of Earth, and very small: 20m since 1900)

The two precession effects are lumped together into the *general precession*. The annual general precession is given by $50.2564 + 0.0222T$ arc seconds, where $T$ is the time in Julian centuries from 1900.00
Two different positions, now (1) and 13,000 years in the future (2).
Effects of Precession of Equinoxes

• Since the location of the equinox changes with time, coordinate systems that are defined by the vernal equinox must have a date associated with them. This specified year is called the Equinox (not epoch).

• The difference between epoch and equinox is that the equinox addresses changes in the coordinate system, while the epoch addresses changes in the position of the celestial body itself.

• The common practice is to use standard years (like 1950, 2000, 2050) to cite the Equinox. Currently we use Equinox J2000.0.
  
  – B1950.0 - the equinox and mean equator of 1949 Dec 31st 22:09 UT. (B refers to Besselian year, years are based on a solar year)
  – J2000.0 - the equinox and mean equator of 2000 Jan 1st 12:00 UT (J refers to Julian year which is 365.25 days)
  – Converting from B1950 to J2000.0 (and vice versa)

    Apply corrections for differences between reference systems, precession & proper motion
Nutation

- **Nutation** is a small cyclical motion superimposed upon the 26,000-year precession of the Earth’s axis of rotation. It is mainly caused by the gravitational attraction of the Sun and Moon, which continuously change location relative to each other and thus cause nutation in Earth's axis. The largest component of Earth's nutation has a period of 18.6 years, the same as that of the precession of the Moon's orbital nodes.

- Because the dynamics of the planets are so well known, nutation can be calculated within seconds of arc over periods of many decades.
Parallax (above) is the geometric effect due to the placement of the earth around its orbit, while aberration of light (right) refers to the apparent change in direction toward a star due to the finite speed of light, plus the earth’s orbital motion.
Atmospheric Refraction

- **Atmospheric refraction** is the deviation of light (or other electromagnetic radiation) from a straight line as it passes through the atmosphere due to the variation in air density as a function of altitude.

- Atmospheric refraction causes astronomical objects to appear higher in the sky than they are in reality. This effect even lifts objects from up to 35 arc minutes below horizon to above the apparent horizon.

The index of refraction, $n$, is the ratio of the speed of light in a vacuum, $c$, to the speed of light in the medium, $v$

$$n = \frac{c}{v}$$
Some Important Astrometrical Programs

- Carte du Ciel
- Hipparcos
- VLBI
- GAIA
A big advance required a new approach - Hipparcos. The satellite spun (2 hours for the entire sky) and its telescope compared two directions separated by about 50 degrees.
The telescope is a reflective Schmidt design, with the corrector split to divide the aperture between the two fields.
Upper - Hipparcos modulating grid, spacing 8.2 \( \mu \text{m} \) projecting to 1.208 arcsec on the sky.

Lower - star mapper used for the Tycho catalog, determining positions with accuracy about 25 times lower than through the grid, and measuring B, V photometry of them.

Satellite spinning caused stars to pass over these masks quite rapidly. The signal modulated by the mask was used to determine star positions.
An image dissector scanner was used with its sensitive area where a star was expected. It was read out 1200 times/second, that is, every 0.15”
VLBI is a geometric astrometrical technique

VLBI measures the time difference between the arrival at two Earth-based antennas of a radio wave-front emitted by a distant quasar (or other source). Using large numbers of time difference measurements from many quasars observed with a global network of antennas, VLBI determines the inertial reference frame defined by the quasars and simultaneously the precise positions of the antennas.

Because the time difference measurements are precise to a few picoseconds, VLBI determines the relative positions of the antennas to a few millimeters and the quasar positions to fractions of a milliarcsecond.
GAIA: ESA Astrometry Space Telescope:

Gaia’s goals:

• measure the positions of ~1 billion stars both in our Galaxy and other members of the Local Group, with an accuracy down to 20 μas

• perform spectral and photometric measurements of all objects

• Make the largest most precise 3-D map of the Galaxy by surveying 1% of its population (1 billion stars)
Gaia will operate in a Lissajous-type orbit, around the L2 point of the Sun-Earth system, which is located 1.5 million km from the Earth in the anti-Sun direction. The orbit is not impacted by Earth eclipses. The orbit period is about 6 months. An operational lifetime of 5 years is planned.
Gaia’s payload consists of a single integrated instrument that comprises 3 major functions:

1. The Astrometric instrument is devoted to star angular position measurements, providing
   - Star position
   - Proper motion
   - Parallax (distance)

2. The Photometric instrument provides continuous star spectra in the band 320-1000 nm

3. The Radial Velocity Spectrometer (RVS) provides radial velocity and high resolution spectral data in the narrow band 847-874 nm

**CCD Focal Plane: 106 CCDS**

- 4 CCDs: Metrology & Alignment (Green: wave front sensors, Yellow: basic angle monitoring)
- 14 CCDs: Initial Star Detection (Strips SM1 & SM2)
- 62 CCDs: Astrometric Field (AF1-AF9) used for precise position measurements
- 14 CCDs: Photometry, 7 Blue CCDs (BP) spectral measurements in 330-680 nm & 7 Red CCDs (RP) spectral measurements in 640-1000 nm.
- 12 CCDs: Spectroscopy (RVS1-RVS3) 847-874 nm
GAIA's Observing strategy

Gaia’s measurement principle relies on the systematic and repeating observation of the star positions in two fields of view. For this purpose, the spacecraft is slowly rotating at a constant angular rate of 1° per minute around an axis perpendicular to those two fields of view, which thus describe a circle in the sky in 6 hours. With a basic angle of 106.5° separating the astrometric fields of view, objects transit in the second field of view 106.5 minutes after crossing the first one.

The orientation of the spin axis is modulated by a slow precession around the Sun-to-Earth line with a period of 63.12 days than enables the observation of about 70 transits of each of the 1 billion stars.
## Gaia: Complete, Faint, Accurate

<table>
<thead>
<tr>
<th></th>
<th><strong>Hipparcos</strong></th>
<th><strong>Gaia</strong></th>
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</thead>
<tbody>
<tr>
<td><strong>Magnitude limit</strong></td>
<td>12</td>
<td>20 mag</td>
</tr>
<tr>
<td><strong>Completeness</strong></td>
<td>7.3 – 9.0</td>
<td>20 mag</td>
</tr>
<tr>
<td><strong>Bright limit</strong></td>
<td>0</td>
<td>6 mag</td>
</tr>
<tr>
<td><strong>Number of objects</strong></td>
<td>120 000</td>
<td>26 million to V = 15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>250 million to V = 18</td>
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<tr>
<td></td>
<td></td>
<td>1000 million to V = 20</td>
</tr>
<tr>
<td><strong>Effective distance</strong></td>
<td>1 kpc</td>
<td>50 kpc</td>
</tr>
<tr>
<td><strong>Quasars</strong></td>
<td>1 (3C 273)</td>
<td>500,000</td>
</tr>
<tr>
<td><strong>Galaxies</strong></td>
<td>None</td>
<td>1,000,000</td>
</tr>
<tr>
<td><strong>Accuracy</strong></td>
<td>1 milliarcsec</td>
<td>7 µarcsec at V = 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 – 25 µarcsec at V = 15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>300 µarcsec at V = 20</td>
</tr>
<tr>
<td><strong>Photometry</strong></td>
<td>2-colour (B and V)</td>
<td>Low-res. spectra to V = 20</td>
</tr>
<tr>
<td><strong>Radial velocity</strong></td>
<td>None</td>
<td>15 km/s to V = 16-17</td>
</tr>
<tr>
<td><strong>Observing</strong></td>
<td>Pre-selected</td>
<td>Complete and unbiased</td>
</tr>
</tbody>
</table>
GAIA: Current Status

- Launched ~ 1000 days age
- First catalog released a month ago (September 14)
  - 14 months of data, up to September, 2015
  - $1.142 \times 10^9$ stars
  - Parallaxes and proper motions for 2 million of them (20 times as many as those with accurate Hipparcos coordinates)
  - Current positions are about twice as accurate as those of Hipparcos
  - That is, there is a long way to go on the analysis, as well as more data to ingest

- Main purpose of initial release was to show high quality of the data
- Clearly in another year or two, this will be the “go-to” astrometric reference
- A side benefit: proper motions mean that the Hipparcos coordinates are less useful than they used to be, so just having new ones will be great by itself
Reference Systems & Frames

The concept of position is a relative one. One can define a position only with respect to something. In astronomy that is a reference system, which is a theoretical concept, or a reference frame, a practical realization of a reference system.

- A **reference system** is the complete specification of how a celestial coordinate system is to be formed. It defines the origin, fundamental planes, it specifies all of the constants, models and algorithms used to transform between observable quantities and the reference data that conform to the system.

- A **reference frame** is the practical realization of the reference system, using as a catalog of positions and proper motions (if measurable) of stars or extragalactic objects.

The fundamental plane has conventionally been the celestial equator and vernal equinox is the zero point for right ascension. The theories of the motion of the Earth that define how the ecliptic and equatorial planes move are imperfect. The exact location of the vernal equinox is difficult to determine.
Reference Frames

The establishment of celestial reference frames is coordinated by the International Astronomical Union (IAU).

  
  – A catalog of 1535 bright stars (to magnitude 7.5), supplemented by a fainter extension of 3117 additional stars (to magnitude 9.5)
  – Complied from meridian observations taken in the visual band
  – The catalogs list mostly nearby stars so any definition of coordinates tied to these catalogs is subject to errors due to motions of stars on the sky.

• **Success of precise wide-angle astrometry with radio observations, using the techniques of Very Long Baseline Interferometry (VLBI) -> new system**
  
  – Uncertainties in radio source positions listed in all-sky VLBI catalogs are typically less than 1 milliarcsecond (and often a factor to 10 better)
  – These radio sources are very distant extragalactic objects (mostly quasars) that are not expected to show measurable intrinsic motion.
  – VLBI observations revealed that models of the Earth’s precession and nutation (that were part of the FK5 system) were inadequate for modern astrometric precision. (The “constant of precession” has been over-estimated by about 0.3 arc seconds per century)
  – Success of Hipparcos astrometric satellite (launch 1989) lead to a new set of very accurate star coordinates in the optical regime.
International Reference System (ICRS)

The ICRS is the fundamental celestial reference system adopted by the IAU in 1997.
• Axes are fixed with respect to a set extragalactic objects which are assumed to have no measurable proper motion.
• The International Reference Frame (ICRF) is the realization of the ICRS

Based on radio positions of 608 extragalactic sources observed with the VLBI distributed over the entire sky.
  – 212 of the 608 sources are defining sources that establish the orientation of the ICRS axes, with the origin at the solar system barycenter.
  – The positional uncertainty of the defining sources are on the order of 0.5 milliarcsecond.
  – The axes correspond closely to the “equator and equinox” of J2000.0
  – The ICRS is realized at optical wavelengths by stars in the Hipparcos Catalog (118,218 stars, some as faint as 12). Only stars with well-determined proper motions are used for the ICRS realization (85%)
ICRS continued

• A new set of standard algorithms were adopted in 2000 to transform the ICRS catalog data to observable quantities (and vice versa). These include precession-nutation model, a new definition of the celestial pole, and a new reference point for measuring the rotation angle of the Earth around its instantaneous axis.
Reference Star Catalogs

http://ad.usno.navy.mil/star/star_cats_rec.html

Star catalog contain a list of stars according to position and magnitude and in some cases other properties (e.g. proper motion, spectral type, and parallax).

- **Hipparcos** contains positions, proper motions, parallaxes, and B and V magnitudes for 118,218 stars. Complete to V = 7.3 Positional accuracy, 1-3 mas at epoch 1991.25. Proper motion accuracy around 1 to 2 mas/yr.

- **UCAC2**: (Preliminary catalog, 86% of sky, 48 million stars in the R = 8.0 to 16.0 magnitude range. Contains positions (20 to 70 mas) and proper motion (1 to 7 mas/yr)

- **TYCHO-2** contains position, proper motions and two-color photometric data for 2,539,913 stars. The catalog is 99% complete to V = 11.0. Positional accuracies 10 to 100 mas and proper motions accuracies from 1 to 3 mas.

- **USNO B 1.0** a catalog of over 1 billion objects. The Tycho-2 catalog is the astrometric reference. All-sky coverage down to V = 21, 200 mas accuracy at J2000, 0.3 magnitude photometric accuracy in up to five colors, and 85% accuracy for distinguishing stars from non-stellar objects.

- **2MASS**- Two Micron All Sky Survey. Point source catalog contain 470 million objects, mostly stars. The extended source catalog contains data on 1.6 million objects. Observations on three photometric bands (JHK). The positions accuracy is about 70 mas. No proper motions.

Astrometry and your images

Few people want to deal with the details of positional astronomy, but many people want to know the precise position of objects they have observed.

There are many software tools to aid you in determining an astrometric solution for your data (IRAF, WCS tools, routines in IDL).

But first we need discuss the world coordinate system and FITS images.
In the late 1970's astronomers developed, the Flexible Image Transport System, **FITS**, as an archive and interchange format for astronomical data files.

In the past decade FITS has also become the standard formats for on-line data that can be directly read and written by data analysis software.

FITS is much more than just an image format (such as JPG or GIF) and is primary designed to store scientific data sets consisting of multidimensional arrays and 2-dimensional tables containing rows and columns of data.
A FITS File

A FITS file consists of one or more Header + Data Units (HDUs), where the first HDU is called the "Primary Array".

- The primary array contains an N-dimensional array of pixels. This array can be a 1-D spectrum, a 2-D image or a 3-D data cube.
- Any number of additional HDUs, called "extensions", may follow the primary array.
- Every HDU consists of an ASCII formatted "Header Unit" followed by an optional "Data Unit". Each header unit consists of any number of 80-character records which have the general form:

```
KEYNAME = value / comment string
```

The keyword names may be up to 8 characters long and can only contain upper-case letters, the digits 0-9, the hyphen, and the underscore character. The value of the keyword may be an integer, a floating point number, a character string, or a Boolean value (the letter T or F). There are many rules governing the exact format of keyword records so it usually best to rely on a standard interface software like CFITSIO, IRAF or the IDL astro library to correctly construct or parse the keyword records rather than directly reading or writing the raw FITS file.
For a complete description of the FITS/WCS projections and definitions see:


World Coordinate System

There are a set of FITS conventions that have been defined to specify the physical, or world, coordinates to be attached to each pixel of an N-dimensional image. By world coordinates, one means coordinates that serve to locate a measurement in some multi-dimensional parameter space.

One common example is to link each pixel in an astronomical image to a specific equatorial coordinate (right ascension and declination).

In general the FITS world coordinate system (WCS) of an image is defined by keywords in the FITS header. The basic idea is that each axis of the image has the following keywords associated with it:

- **CTYPEi** Type of coordinate on Axis i, 8 characters (The FITS WCS standard defined 25 different projections which are specified by the CTYPE keyword.)
- **CRPIXi** Reference pixel on Axis i
- **CRVALi** Value of World Coordinate at Axis i at reference point (CRPIXi)
- **Cdi_j** A Matrix of partial derivatives of the World Coordinates with respect to the pixel coordinates.

Old style:

- **CDELTi** coordinate increment on Axis i
- **CROTAi** rotation parameter for each Axis i

\[
\begin{align*}
\text{cd1}_1 &= \text{crpix1} \cos(\text{crota2}) \\
\text{cd1}_2 &= \text{crpix2} \cos(\text{crota2}) \\
\text{cd2}_1 &= -\text{crpix1} \cos(\text{crota2}) \\
\text{cd2}_2 &= \text{crpix2} \cos(\text{crota2})
\end{align*}
\]
\begin{verbatim}
SIMPLE = T / Fits standard
BITPIX = -64 / Bits per pixel
NAXIS = 2 / Number of axes
NAXIS1 = 13600 / Axis length
NAXIS2 = 16700 / Axis length
EXTEND = T / File may contain extensions
ORIGIN = 'NOAO-IRAF FITS Image Kernel July 2003' / FITS file originator
DATE = '2008-10-20T16:11:11' / Date FITS file was generated
IRAF-TLM = '13:21:02 (22/10/2008)' / Time of last modification
COMMENT 'FITS (Flexible Image Transport System) format is defined in 'Astronomy and Astrophysics', volume 376, page 359; bibcode: 2001A&A...376..359H
CTYPE1 = 'RA---TAN'
CTYPE2 = 'DEC---TAN'
CRVAL1 = 217.812837962
CRVAL2 = 34.051007516
CDELT1 = -0.000239631
CDELT2 = 0.000239631
CRPIX1 = 7105.
CRPIX2 = 8073.5
CROTA2 = 0.0
PROGID = 40839
BUNIT = 'MJy/sr'
ZEROPT = 17.997
HISTORY = 'BCD 16.1.0,17.0.4'
HISTORY = 'IRACproc-4.1.2; Schuster et al 2006, SPIE, 6270, 65'
HISTORY = 'Montage-v3.0'
HISTORY = 'Processed for the Spitzer Deep Wide Field Survey team'
HISTORY = 'SDWFS, at the Harvard-Smithsonian Center for Astrophysics'
HISTORY = 'by MLN Ashby, 2008 October 22. Version 3.2'
END
\end{verbatim}

\[
Xi = \text{cdelt1} \times (x - \text{crpix1}) \times \cos(\text{crot}2) - \text{cdelt2} \times (y - \text{crpix2}) \times \sin(\text{crot}2)
\]
\[
Eta = \text{cdelt1} \times (x - \text{crpix1}) \times \sin(\text{crot}2) + \text{cdelt2} \times (y - \text{crpix2}) \times \cos(\text{crot}2)
\]

Or
\[
Xi = \text{cd}1_1 \times (x - \text{crpix1}) + \text{cd}1_2 \times (y - \text{crpix2})
\]
\[
Eta = \text{cd}2_1 \times (x - \text{crpix1}) + \text{cd}2_2 \times (y - \text{crpix2})
\]

Then:
\[
\alpha = \tan \text{ projection (xi,eta)}
\]
\[
\delta = \tan \text{ projection (xi,eta)}
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Practical Usage of the FITS/WCS- matching your data to a reference catalog

There are a number of software packages that aid the astronomer in reading, using, or modifying the astrometric information found WCS parameters of the image header. A few of the most commonly used packages are IRAF, WCStools, WCSLIB, and packages in the astronomy IDL library.

If an adequate WCS does not exist for an image the basic steps are:
1. Read in the FITS image and its header
2. Read in a reference catalog (selecting appropriate parameters)
3. Match the reference stars to the image stars – producing a matched reference catalog, pixel values, $\alpha, \delta$
4. Using one of the above WCS software packages perform a fit between the matched star's pixel and $\alpha, \delta$. (The fit can account for various types of distortions on the image, coma, differential astronomical refraction, and telescope flexure). Write the resulting WCS information to the header.
5. Check the result and perform steps 3 and 4 until satisfied
Example of Creating a WCS

To get an initial WCS you must have at least 3 coordinate pairs (pixel position & coordinates of object)

Improvement – blue using more reference stars (min ~ 12)

Improvement

Verify - Finished