Optical Interferometry
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# Motivations for Interferometry

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History of Astronomical Interferometry

- 1868 Fizeau first suggested stellar diameters could be measured interferferometrically.
- Michelson independently develops stellar interferometry. He uses it to measure the satellites of Jupiter (1891) and Betelgeuse (1921).
- Further development not significant until the 1970s. Separated interferometers were developed as well as common-mount systems.
- Currently there are approximately 7 small-aperture optical interferometers, and three large aperture interferometers (Keck, VLTI and LBT)
Interferometry Measurements

Interferometers can be thought of in terms of the Young’s two slit setup. Light impinging on two apertures and subsequently imaged form an Airy disk of angular width $\lambda / D$ modulated by interference fringes of angular frequency $\lambda / B$. The contrast of these fringes is the key parameter for characterizing the brightness distribution (or “size”) of the light source. The fringe contrast is also called the visibility, given by

Visibility is also measured in practice by changing path-length and detecting the maximum and minimum value recorded.

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$
Optical vs. Radio Interferometry

Radio interferometry functions in a fundamentally different way from optical interferometry.

Radio Telescope arrays are *heterodyne*, meaning incoming radiation is interfered with a local oscillator signal before detection. The signal can then be amplified and correlated with signals from other telescopes to extract visibility measurements.

Optical interferometers are *homodyne*, meaning incoming radiation is interfered only with light from other telescope. This requires transport of the light to a central station, without the benefit of being able to amplify the signal. However, homodyne interferometry allows large bandwidths to be used since the interfered light is detected directly.

(One heterodyne optical interferometer (ISI) has been built to operate at 10 microns. The technique is feasible, but limited to bright sources.)
The image brightness distribution of an object is related to its mutual coherence (or visibility) through an imaging system by the van-Cittert Zernike theorem. The essence of this theorem is that the brightness distribution of an object is related to its visibility by a Fourier transform.

\[ \nu (u, v) = \frac{\iint I(l, m) e^{-i2\pi (ul + vm)} \, dl \, dm}{\iint I(l, m) \, dl \, dm} \]

\[ V = |\nu| \]

The visibility is calculated for a given value of the baseline. The baseline is expressed in a coordinate system (u,v,w space) where the w-axis points at the source, the v-axis points towards the north and the u-axis points east.

To compare this complex number to the visibility as defined by Michelson we take the modulus of the complex visibility.

Relating Entrance Aperture to the PSF

Entrance Aperture/Array Baseline

Convolution \downarrow

Modulation Transfer Function/UV-plane coverage

\begin{align*}
\text{Fourier Transform} & \quad \leftrightarrow \\
\text{Multiplication} & \quad \downarrow \\
\text{PSF/Response Intensity} & \quad \leftrightarrow
\end{align*}
The van Cittert-Zernike theorem, applied to single aperture image formation yields the Airy pattern response for a point source.
Two Element Relation

For many interferometers \( b \gg D \) so that the spatial frequencies measured within an aperture are much less than those measured between apertures. For this situation each element is treated as a point in the entrance aperture.
There are $n(n-1)/2$ points in the $u$-$v$ plane for an $n$ element interferometer.

For multiple element array the best choice of spacing is to create uniform sampling of the $u$-$v$ plane. The uniform sampling of spatial frequencies helps avoid degeneracies or aliasing in reconstructing an image.
LBT: an intermediate case

The LBT has a 14.4 m separation and 8.4 elements. Thus in the direction of the baseline all spatial frequencies are sampled from 0 to 22.8 m. This can be seen in the uv-plane coverage for the LBT.
Tracking a source with an interferometer

The baseline of an interferometer, as projected onto the u-v plane, is not fixed as the telescope tracks the object across the sky. For independently mounted telescopes both the baseline length and direction changes. For co-mounted (or co-moving) elements only the baseline orientation changes.

This change in baseline allows measurements over different spatial frequencies and directions, filling in the u-v plane for better image synthesis.

Animation of the LBT tracking a source at declination 20 degrees from hour angle -3 to +3
The LBT tracking a source at d=20

The LBT will not spend equal time at each orientation during observations. This will result in increased sidelobes in the final PSF compared to optimal sampling of the u-v plane.
The LBT tracking a source at d=0

At lower declinations the LBT will be unable to sample all orientations.
Tracking a source with separated elements

Separated-element interferometers trace out elliptical curves in the u-v plane as they track sources across the sky.
Spatial information for long baseline two-element interferometers is primarily in one direction.
Keck Interferometer response for declination=0

At declination 0 the elliptical curves become straight lines in the u-v plane.
Types of Interferometers

Pupil-Plane

Pupil-plane interferometry is used in long-baseline interferometry. Bracewell (1978) first suggested using this technique to null a stellar point source for detection of planets.

Image-Plane

Imaging interferometry is more typically implemented on a common-mount interferometer. An imaging interferometer can be designed to create high resolution images over a wide field of view.
The Keck Interferometer is made up of two independent telescopes. Beam transfer optics and delay lines allow it to maintain equal path-length between the telescope as the object is tracked across the sky. Since the baseline pupil geometry changes no attempt is made to preserve the sine condition.

The LBT is a single-mount structure resulting in a fixed entrance pupil. This allows straightforward implementation of wide-field imaging.
The Sine Condition

Properly designed imaging systems obey the sine condition for the relation of the object plane to the image plane. For imaging systems with the object at infinity the relation becomes

$$\sin \alpha = \frac{h}{f}$$

where $h$ is the height of the ray from the optical axis and $f$ is the focal length of the system.

For interferometers, obeying this design constraint results in interference fringes for a source anywhere in the focal plane.

For interferometers not obeying this constraint the field is much smaller.
What is seen in the focal plane?

Pupil-Plane

- An image of the object is formed with a resolution of $\lambda/D$.
- Visibility is measured by scanning the interferometer in pathlength.
- The flux seen in the image is the object flux multiplied by the transmission pattern, and convolved with the single aperture PSF.

Image-Plane

- An image of the object is formed with a PSF which is the FT of the entrance aperture.
- PSF is invariant within the field.
- The flux seen in the image plane is the object flux convolved with the PSF of the interferometer.

Coherence length

$$l = \frac{\lambda^2}{\Delta\lambda}$$
Pathlength correction in Interferometry

**Pupil-Plane Interferometry**

- A fast scan (~10 ms) of the fringe packet is always being carried out.
- Fitting the fringe packet allows correction of phase errors.
- Functionally equivalent to phase shifting interferometry.

**Image-Plane Interferometry**

- The fringe centroid is calculated and fed back to stabilize pathlength error.
- Functionally equivalent to tip correction on an AO system.
- Danger of jumps in pathlength by one wavelength.
Phase Tracking

For both types of interferometers the phase between the apertures must be sensed and corrected at a rate well above the coherence time of the atmosphere. This typically requires 100-1000 Hz rates for detection of the phase. If we assume phase sensing is done in K band (2.0-2.4 microns) an 8 m aperture receives ~150 photons from a K=15 star in 1 ms. Roughly, phase can be determined to a precision of

$$\delta x = \frac{\lambda}{B \sqrt{SNR}}$$

If we require knowledge of phase to 1/10 the fringe width we require an SNR~10 which is roughly what we have for a perfect detector in K band after 1 ms. In reality we probably lose a couple magnitudes to detector noise, and throughput losses. So we require a star similar in brightness to that needed for AO correction of the individual apertures.
Observations with Interferometers

So what is actually measured?
How is it used?

Visibility → Size of objects
Astrometry → Position of objects
Imaging → Structure of objects
Nulling → Detection of faint objects
Visibility measurements

Alpha Bootis 550 nm

4000 K, log g=1.5, [A/H]=0

21.1 mas

Quirrenbach 1996

21.1 mas

Actual starlight fringes from IOTA - β And

Photo credit: R.R. Thompson
Shao and Colavita (1992) analyze astrometric precision in the presence of turbulence. They derive that for measuring the position relative to a reference star, distance $\theta$ away, using an interferometer with baseline $B$:

$$\delta \chi \propto \frac{\theta}{B^{2/3}}$$

- Want baseline as long as possible without resolving the star
- Need reference stars as close as possible
- Results in expected precisions of 10-30 uas in an hour (capable of detecting planets down to the mass of Uranus for nearby stars)

Interferometers measure angular offset as a pathlength difference between interference fringes.

For angular separations above ~0.5” this requires a dual-star interferometer (!)
Imaging (VLTI)

The European Southern Observatory runs the Very Large Telescope project, with 4 large (8 m) telescopes and 3 smaller telescopes (1.8 m) capable of interferometry of various types and complexity.
VLTI u-v plane coverage

The uv-plane

This is the uv-plane:

Note: This is the uv-plane for an object at zenith. In general, the projected baselines have to be used.
VLTI u-v plane coverage

The uv-plane with the UTs

uv coverage for object at -15°
8 hour observation with all UTs

Resulting PSF is the Fourier transform of the visibilities
\[ \lambda = 2.2 \mu m \ (K\text{-band}) \]

Need to measure visibility and phase to synthesize image.
Point-Source Suppression (Nulling Interferometry)

Nulling at the MMT

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![Diagram of telescope axis and ambient temperature optics with labels M1, M2, M3, Z1, Z2, and A, B.]

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**α Tau**

- Single 50 msec. exposures with 10 μm camera
- For unresolved star, destructive/constructive peak ratio = 0.04

**α Ori**

- For α Ori, peak ratio = 0.18
- Residual flux is a direct thermal image of the extended dust nebula

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**Infrared Detector**
The fundamental limit to suppression for a perfect nulling interferometer is set by the size of the stellar disk, relative to the fringe spacing. The effect limits nulling to $2 \times 10^{-5}$ for typical stars at the LBT. This effect scales as the baseline squared.
Suppression Level

The suppression level for an interferometer used as a nulling instrument is governed largely by the performance of the adaptive optics system. Residual aberrations in the wavefronts causes some of the light to not interfere exactly out of phase. The level and distribution of this light can be estimated by estimating what the wavefront errors are in the presence of atmospheric turbulence and adaptive optics correction.

Spatial Fitting Error

\[
\sigma_{\text{fit}} = \sqrt{2 \cdot 0.55 \left( \frac{\Delta x}{r_0} \right)^{5/6}}
\]

Temporal Lag Error

\[
S_{\text{fit}} = e^{-\sigma_{\text{fit}}^2}
\]

Fried length

actuator spacing

Residual Intensity (N) from these phase variations:

\[
N_{\text{fit}} = \frac{1 - S_{\text{fit}} \left( \frac{\Delta x}{D} \right)^2}{2}
\]

For \( r_0 = 6 \) m, a spacing of 0.5 m gives

\( N_{\text{fit}} = 2 \times 10^{-5} \)

Temporal Lag Error

\[
\sigma_{\text{time}} = \sqrt{2 \cdot 5.5 \left( \frac{\Delta t}{t_0} \right)^{5/6}}
\]

\[
S_{\text{time}} = e^{-\sigma_{\text{time}}^2}
\]

\[
N_{\text{time}} = \frac{1 - S_{\text{time}} \left( \frac{r_0}{D} \right)^2}{2}
\]

For \( v = 6 \) m/s, and a lag of 1 ms,

\( N_{\text{time}} = 2 \times 10^{-4} \)

Temporal lag errors dominate nulling interferometry
The Large Binocular Telescope

- 8.4 m
- 14.4 m

LBTI

6.5 m MMT
LBTI on the LBT
July 14, 2008
beamcombiner

8.4 m LBT primary
14.4 m separation
f/15 telescope foci
off-axis ellipse reimagers

right focal plane
left focal plane
combined focal plane
f/15 envelope, f/41.2
individual beams
40 arcsec field

Beam Combination

PZT mounted mirror for fast tip, tilt and phase compensation
pupil image for cold baffling
adjustable mirror for tip, tilt, and path adjustment

3.8 m
UBC Optical Performance

Interferometric Performance analyzed by a custom ray trace code (developed by C. Peng)

• "Perfect" on-axis imaging

• Design delivers >80% Strehl over a 40" diameter field at 2.2 microns.

• Pupil image at fold mirror allows precise cold stops

• Design study determined this was the optimum three mirror design
LBTI design

- Discrete cold dewars
- External Rigid Structure
- General Purpose (Universal) Beam Combiner (UBC)
- Three Camera Ports
- Nulling and Imaging Camera (NIC) is the only funded camera
- Integrated Wavefront Sensors
LBTI on the LBT
### Strengths of the LBT Interferometer

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<th>Characteristic:</th>
<th>Imaging Interferometry</th>
<th>Nulling Interferometry</th>
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<tr>
<td>Fixed-Pupil</td>
<td>Simple wide-field beam combination</td>
<td>Small number of reflections (low emissivity)</td>
</tr>
<tr>
<td>Modest Baseline</td>
<td>Complete uv-plane coverage</td>
<td>High suppression of starlight (stellar disk not resolved)</td>
</tr>
<tr>
<td>Deformable Secondary</td>
<td>Simple beam train</td>
<td>Keeps warm elements to a minimum.</td>
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<tr>
<td>8.4 m apertures</td>
<td>Sensitive to faint objects</td>
<td>Sensitive to faint objects</td>
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LBTI Fizeau Imaging Capabilities

Fizeau imaging snapshot of Io

Reconstruction from three images formed at 60 degree intervals

resolution appropriate for I, J, & H bands

(simulation by Keith Hege)
Auxiliary Material
Nulling Interferometry

Colder dust is easier to see than warmer dust with Spitzer.

Zodiacal dust model from Kelsall 1998 with 0.1 and 40 AU cutoffs

- =15% limit for Spitzer passbands
- =0.1% limit for nulling observations
- =0.01% limit for nulling observations

3000 zodies
300 zodies
30 zodies
3 zodies
The Bracewell Infrared Nulling Cryostat

6.5 m MMT used as a nulling interferometer

Optical layout of the Bracewell Infrared Nulling Cryostat (BLINC)
Nulling Interferometry: Space-based
Nulling Interferometry: Space-based
Why from space?

Space-based telescope gives $>10^6$ reduction in background light. => Collecting area can be $<10^{-3}$ of ground based system.
Effects of Turbulence

The atmosphere limits interferometer performance in several distinct ways. While adaptive optics is helpful in improving performance, atmospheric effects still define the limit for most measurements.

Field of view: Limited by isoplanatic patch for imaging interferometry


Astrometry Precision limited by anisoplanicity of beams.

Nulling AO performance determines suppression level.
Field-of-View Limits

**Pupil-Plane**

For pupil plan combination, the phase difference between the two apertures depends on the sources position on the sky. So a source $\lambda/2B$ away from the main source would have $1/2$ wave phase difference from the main source. Once the source has introduced a phase difference equivalent to the coherence length, the source does not create fringes.

**Image-Plane**

If the sine condition is preserved for image-plane interferometry the FOV is set by the parameters of the atmospheric correction. Stars within the isoplanatic patch will have similar atmospheric phase errors, allowing tracking of phase on a bright star in the field and measurement of fringe contrast for fainter stars anywhere within the isoplanatic patch.

\[
\theta = 0.31 \frac{r_0}{h}
\]

This is approximately 20" for 2.2 microns

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<th>Parameter</th>
<th>Formula</th>
<th>Unit</th>
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<td>Coh. length</td>
<td>$l = \frac{\lambda^2}{\Delta \lambda}$</td>
<td>microns</td>
</tr>
<tr>
<td>Path difference</td>
<td>$\Phi = \frac{\theta}{\lambda/\Delta B} \left(\frac{\lambda}{B}\right)$</td>
<td>microns</td>
</tr>
<tr>
<td>FOV</td>
<td>$\theta = \frac{\lambda^2}{B \Delta \lambda}$</td>
<td>radians</td>
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This is $\sim 0.03"$ for Keck at 2.2 microns